Week 05: Swimming at Low Reynolds Number

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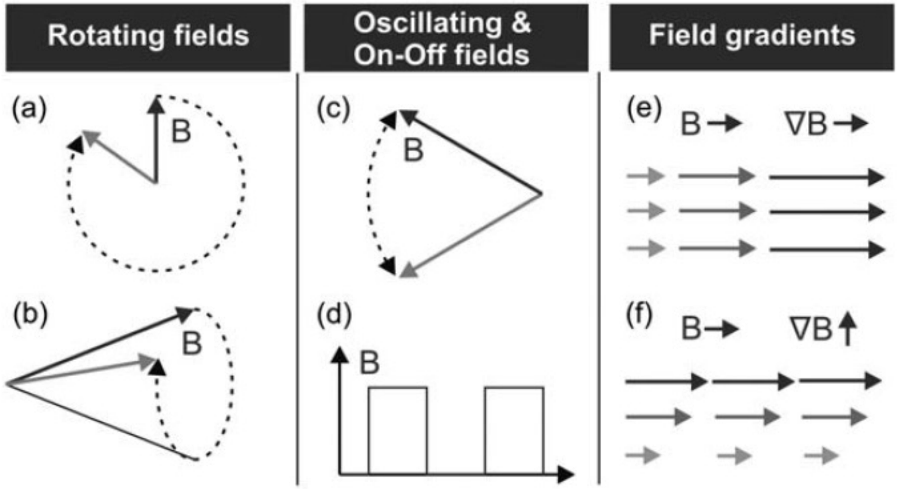
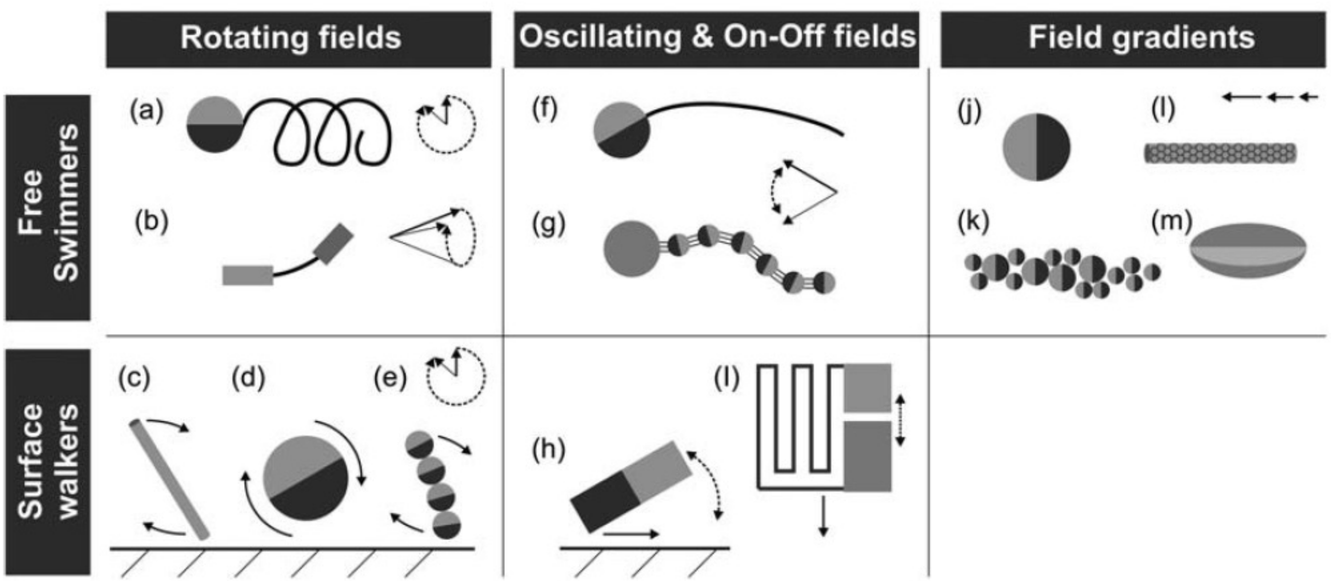
Lecture Overview

- Scallop Theorem
- Bioinspired Swimming Strategies
- Fabrication and Control of Microswimmers

- Next week: Elasto-magnetic Actuators and Machines

Summary of Magnetic Control Methods





Physics of Swimming

- Moving through a fluid is affected by two fundamental phenomena
  - **Inertial effects**: Moving (i.e. accelerating) the fluid away from where we want to be
  - **Viscous effects:** Overcoming the friction between the fluid layers that are moving with us and those that are not

Navier-Stokes Equation

- The Navier-Stokes equations is a formulation of Newton's second law applied to a fluid to describe its motion
  - For an incompressible Newtonian fluid:

unsteady acceleration

Viscosity term

$$\rho\left(\frac{\partial \mathbf{V}}{\partial t}+\mathbf{V}\cdot\nabla\mathbf{V}\right)=-\nabla p+\mu\nabla^2\mathbf{V}$$

Convective acceleration

Pressure gradient

Reynolds Number

- The Reynolds (Re) number is a dimensionless number that describes the relative importance of inertial and viscous effects:

$$\left(\frac{\rho U \delta}{\mu}\right)\frac{d\tilde{\mathbf{V}}}{dt} = -\nabla\tilde{p} + \nabla^2\tilde{\mathbf{V}}$$

$$Re = \frac{\rho U \delta}{\mu}$$

where

ρ	is fluid density	(kg/m^3)
U	is characteristic speed	(m/s)
δ	is characteristic length	(m)
μ	is fluid viscosity	(Ns/m^2)

- Re << 1: Viscous forces dominate Inertial Forces
- Navier-Stokes becomes time-independent (Stokes Flow)

$$\nabla p = \mu \nabla^2 V$$

Intermediate Reynolds Number

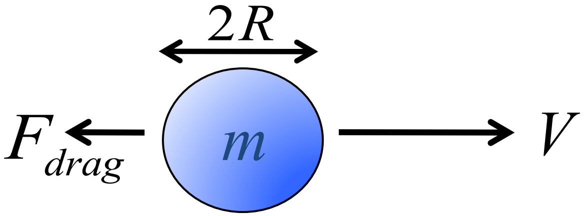
- $1 < Re < 1000$
- Both viscous and inertial effects play an important role.
- Examples:
  - Insect flight
  - Micro aerial vehicles (MAV)

Drosophila melanogaster

Stokes Flow

- Describes the drag force on a sphere in Stokes flow

$$F_{drag} = 6\pi\mu RV$$



- The force is linearly proportional to the radius of the object

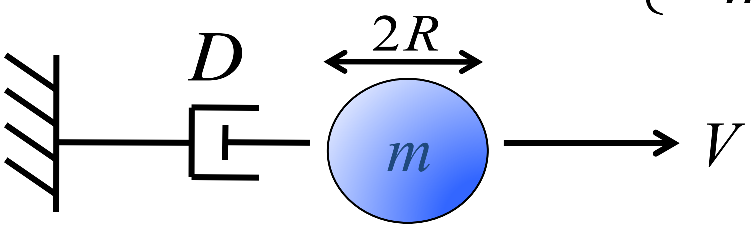
- Microsphere (R = 1 μm, ρ = 10^4 kg/m^3) being pulled through water (μ=10^{-3} Pa·s , ρ = 10^3 kg/m^3) at a speed of V = 10 μm/s

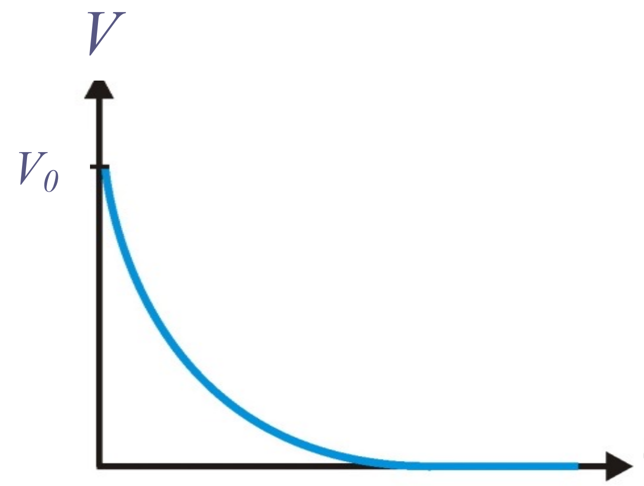
$$F_{pull} = F_{drag} = DV \approx 20\, pN$$

Stokes Flow

- Velocity of the sphere
  - Behaves like a critically damped system

$$V(t) = V_0 \exp\left\{-\frac{D}{m}t\right\}$$





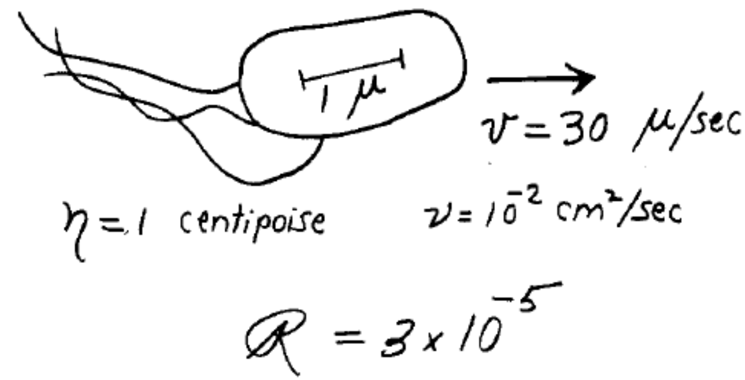
- Coasting distance and time

$$d_{coast} = \int_0^\infty V(t)\,dt = V_0 \frac{m}{D} \approx 2 \cdot 10^{-11} m \qquad t_{coast} \approx 2\,\mu s$$

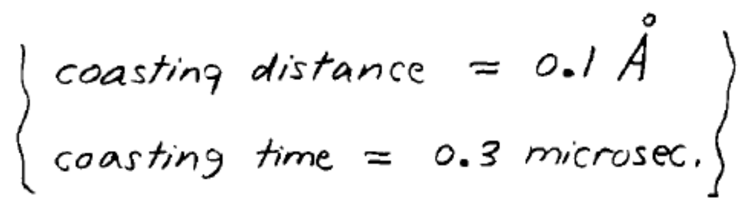
- Coasting distance is only $d_{coast} = 10^{-5}$ of the sphere's radius!
- Steady state is reached almost immediately

Swimming at low Reynolds Number

- Bacteria swim at Re ≈ 10^{-4} in water
- What the bacterium is doing at the moment is entirely determined by the forces that are exerted on it at that moment and by nothing in the past

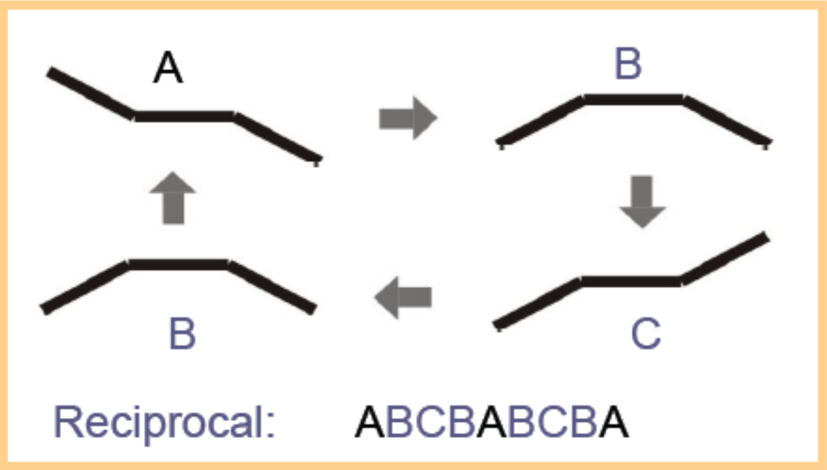


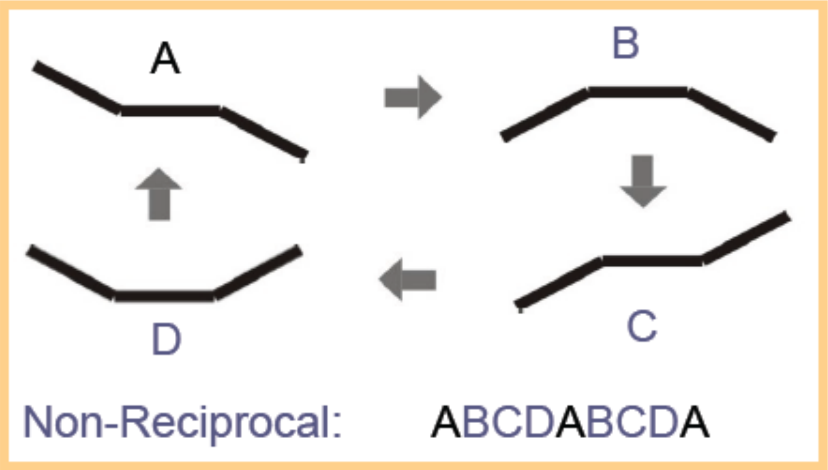
- Reciprocal Motion
  - Time makes no difference
  - If I change quickly or slowly, forward or backwards, the pattern of motion is exactly the same



Swimming at low Reynolds Number

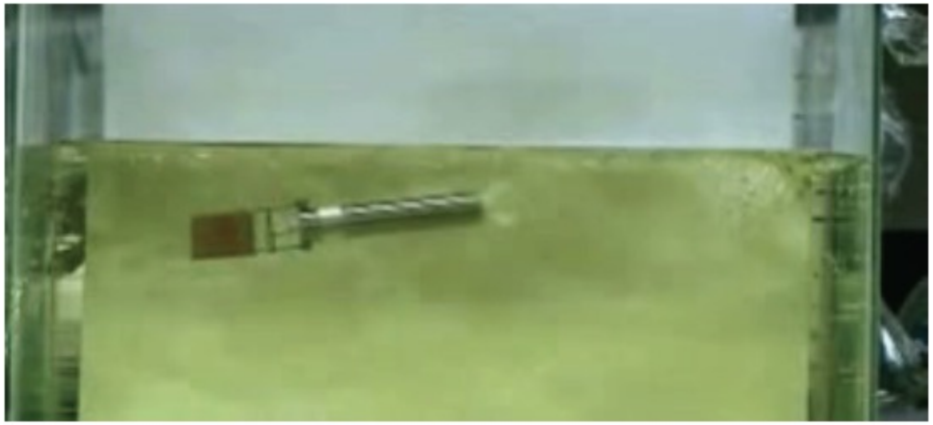
- A micro-swimmer must generate non-reciprocal motion in order to produce a net displacement (in Newtonian fluids)
- More than one degree of freedom is necessary to create non-reciprocal motion
- Example: a swimmer with two hinges
  - Depends on set of configurations



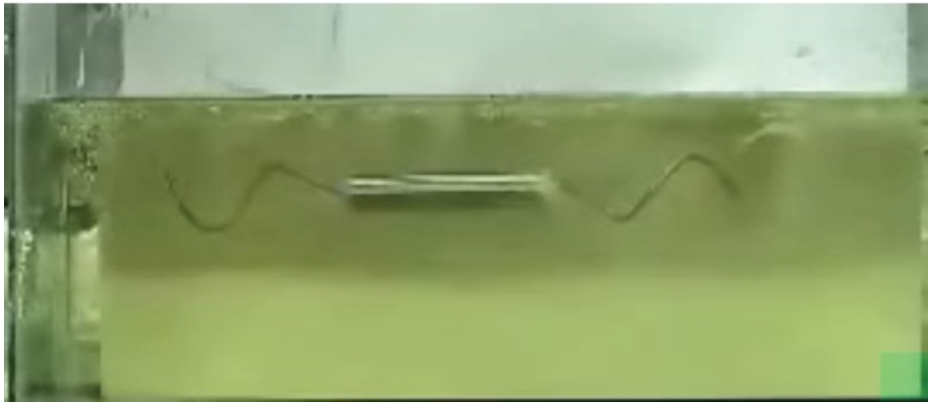


Swimming at low Reynolds Number (video)

- Reciprocal motion
  - No net displacement after one cycle
  - Rigid oar moving left and then right

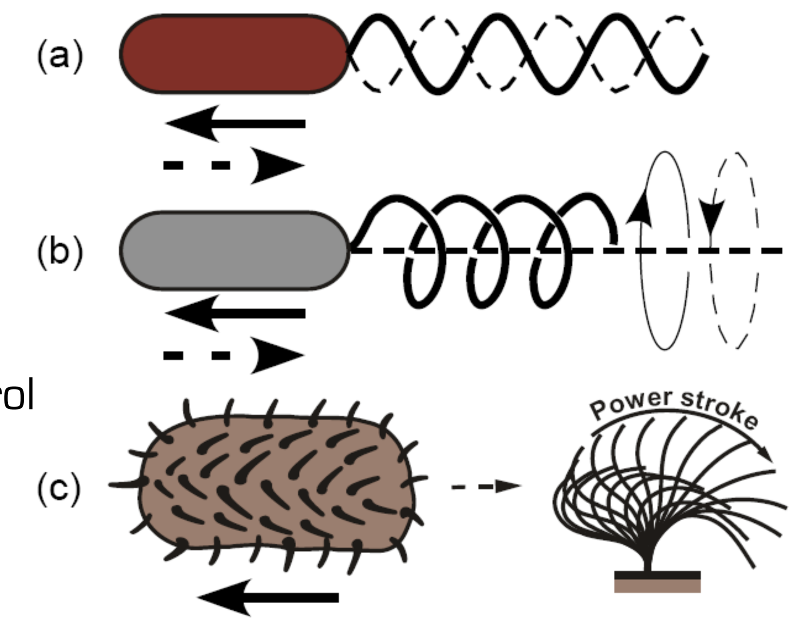


- Non-reciprocal motion
  - Net displacement after one cycle
  - One rotation around helical axis



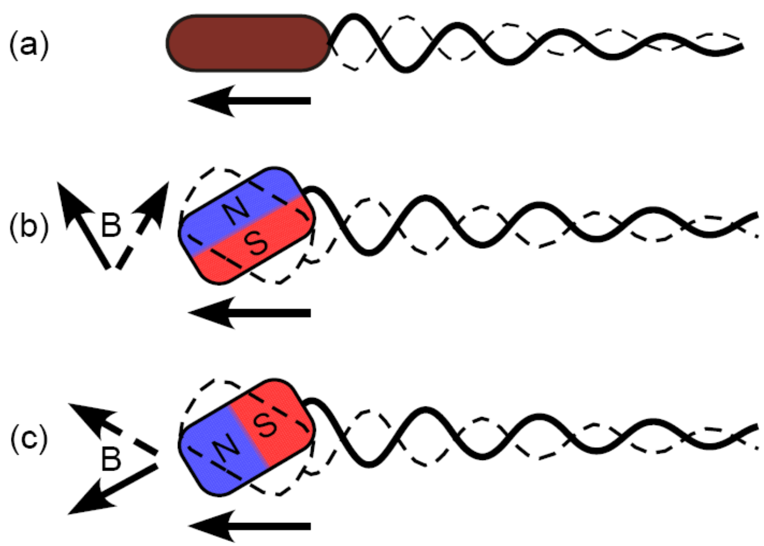
Bioinspired Swimming

- Eukaryotic flagella (a)
  - Active organelles which create traveling waves
  - Swimming direction can be reversed by reversing the direction of the wave
    - Head-to-tail
    - Tail-to-head
- Bacteria flagella (b)
  - Molecular motors turn the flagella
- Cilia (c)
  - Active organelles
  - Held perpendicular during the power strol
  - Parallel during recovery stroke



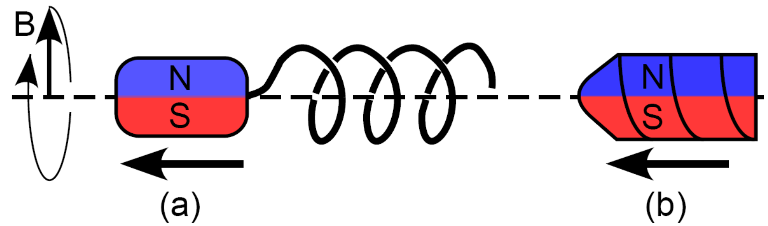
Bioinspired Swimming

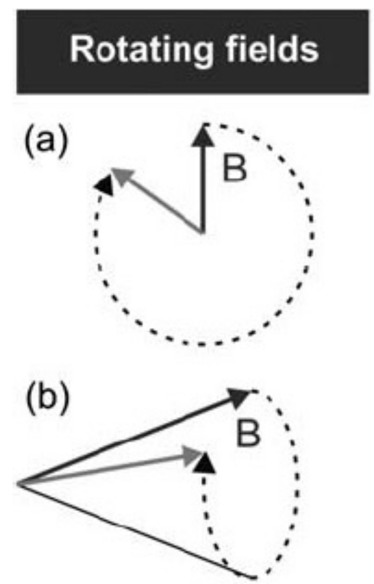
- One-sided actuation
  - "Flexible oar" (a)
    - There exists an optimum in tail elasticity and length
    - Too short & rigid
      - "Scallop theorem"
    - Too long & elastic
      - Increased drag
  - Use of varying magnetic fields (b-c)
    - Magnetic field creates a torque on a magnet
    - By varying the field, the torque is a function of time
    - Induces a waving motion to the following tail



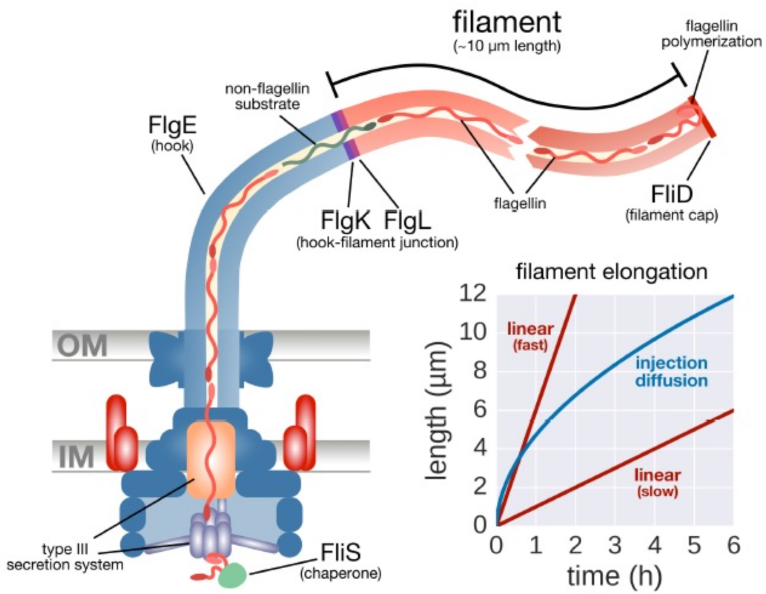
Bioinspired Swimming

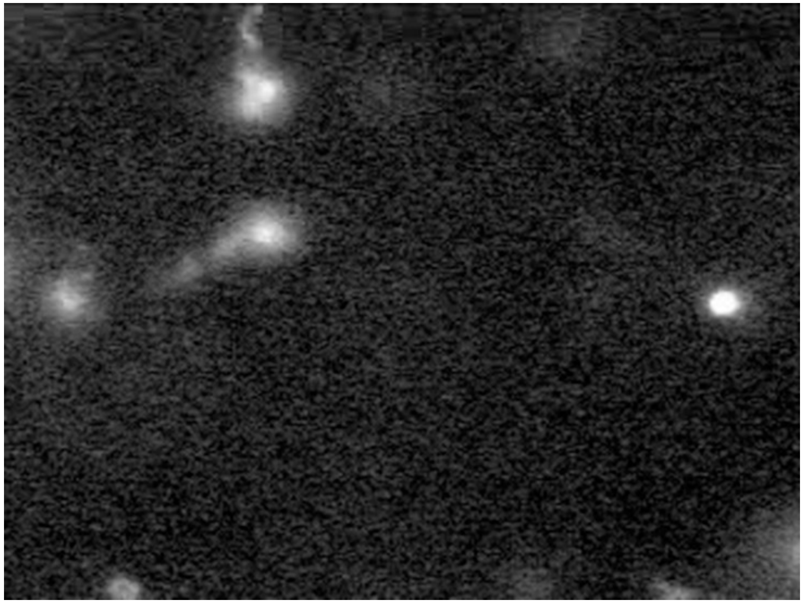
- Another plausible solution is trying to recreate helical swimming, with rotating magnetic fields
- Helical propeller
  - A helical tail can be attached to the "head" of the microrobot
  - The interaction between the magnet and the field causes the magnet to rotate
  - Swimming velocity is linearly related to the field frequency, up until a step-out frequency
    - Velocity decreases dramatically

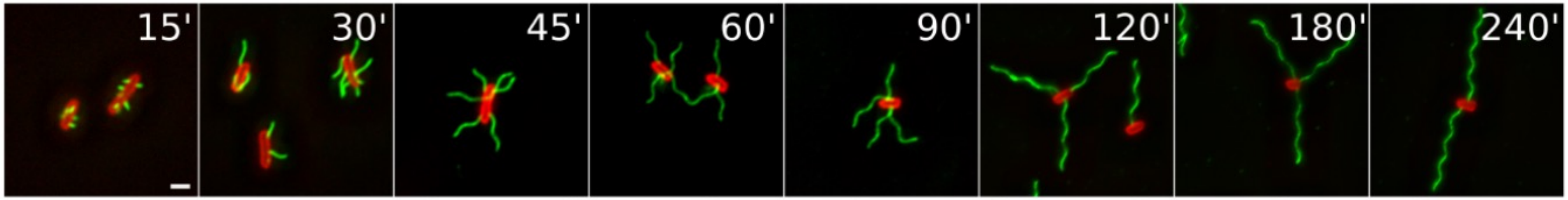


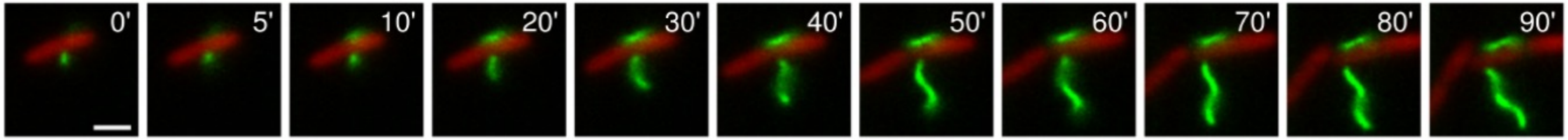


Bacterial Flagella



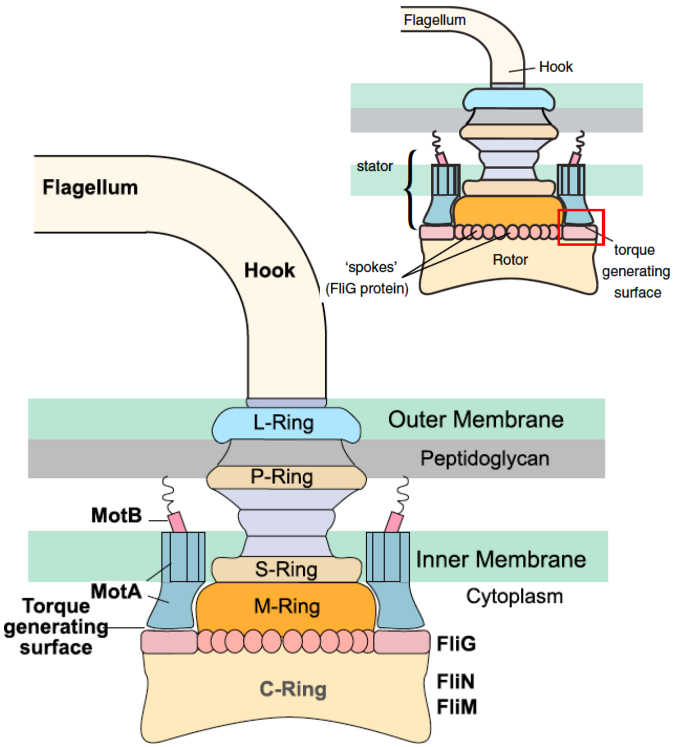


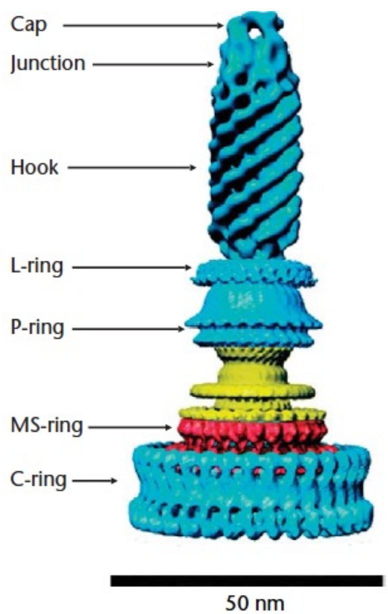


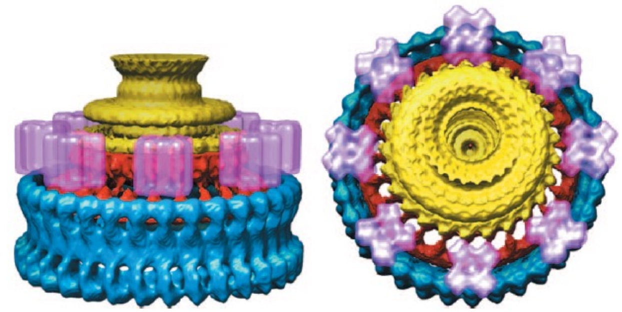


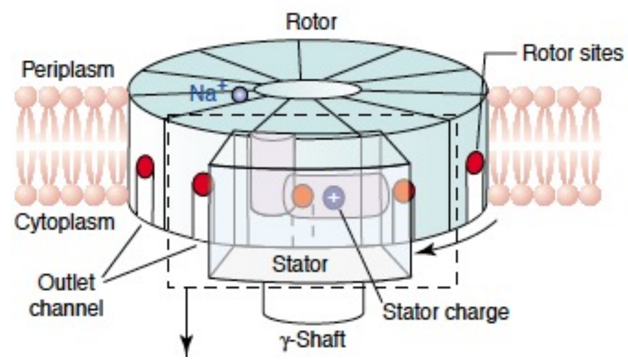
Flagellar Motor

Stator complex (Mot A and Mot B)– Rotor ring (C ring) – Axial driveshaft (rod) – Universal joint (hook) – Helical propeller (filament)







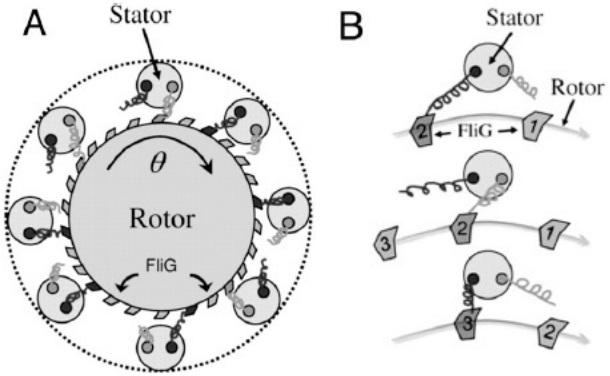


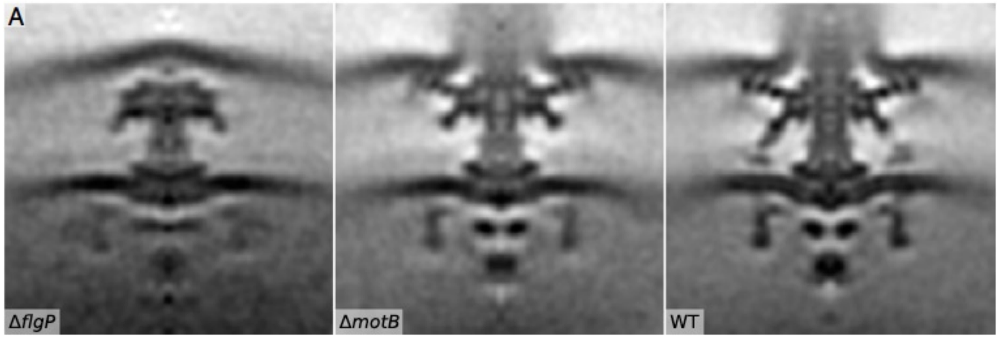
Torque is generated at the interface between transmembrane proteins (stators) and rotor
Passage of ions down a transmembrane gradient through stator complex provides energy
Each revolution 1200 protons, each contributing $6k_BT$, 26 steps per revolution, up to 300Hz

Mechanism of Torque Generation

Torques of motors from different bacteria (torque correlates with swimming speed)

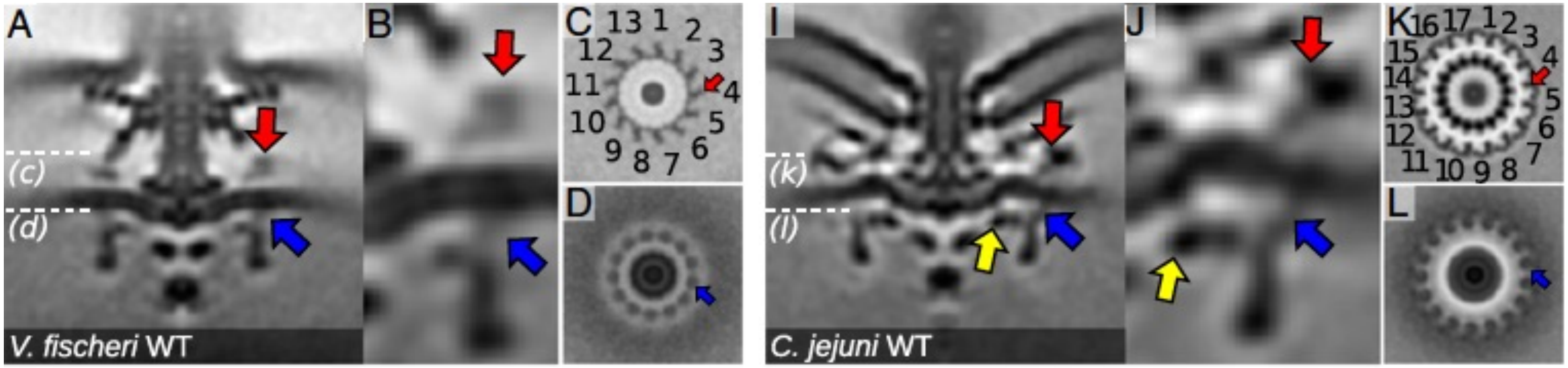
C. Crescentus: 350 pN.nm, E.coli: 2000 pN.nm, H. pylori: 3600 pN.nm, spirochetes: 4000 pN.nm



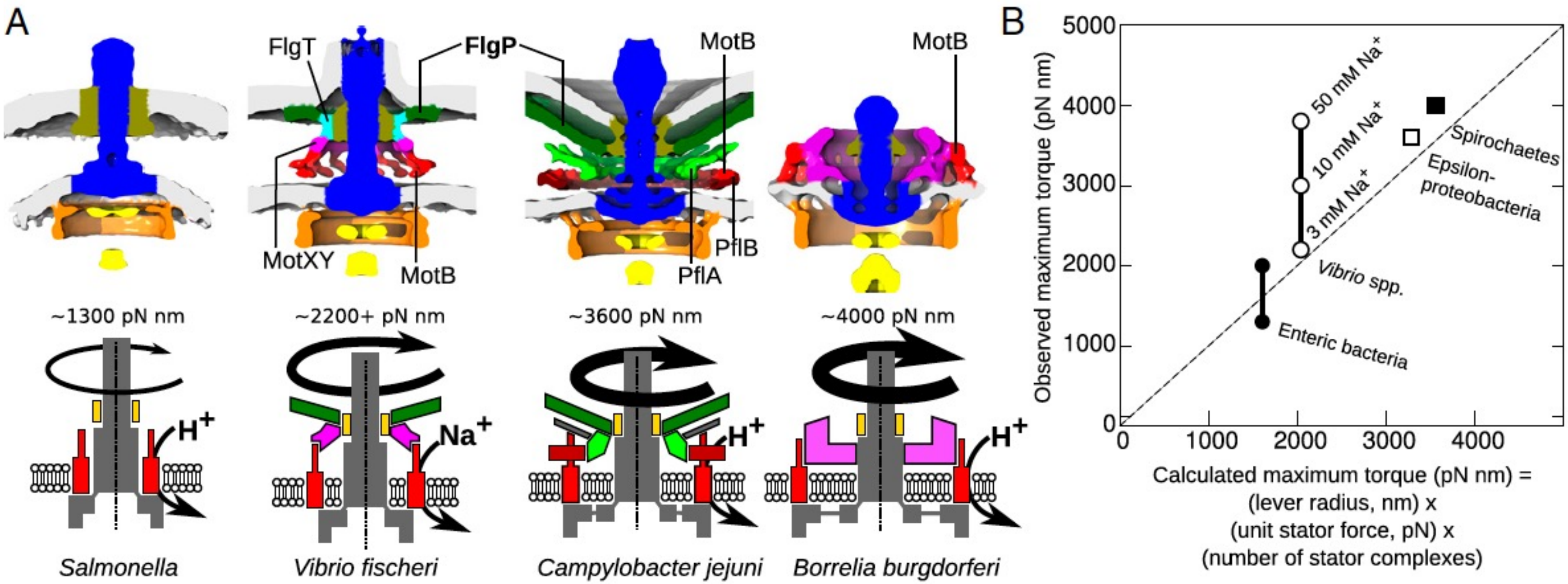


Structural adaptations of flagellar motors

Salmonella: 11 stator complexes, Vibrio: 13, C. jejuni: 17



Mechanism of Torque Generation

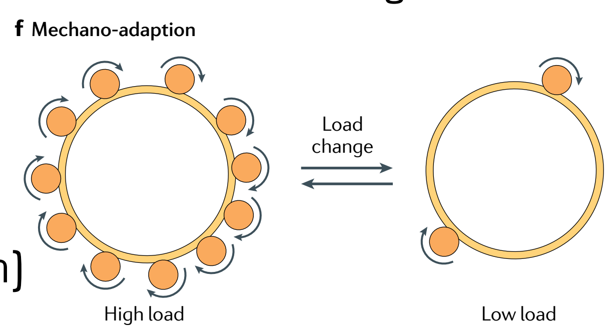


The stator number and increasing wider C-ring in each motor correlate with the generated torque

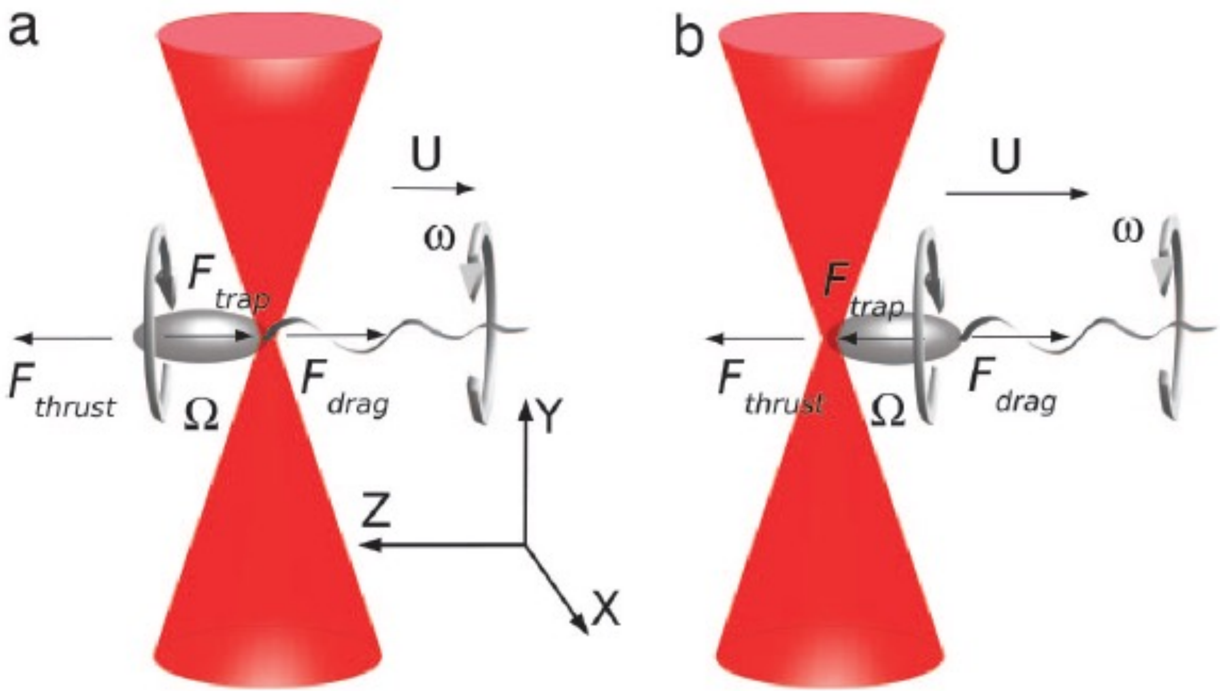
A single stator complex exerts 7.3 pN

Lever contact point

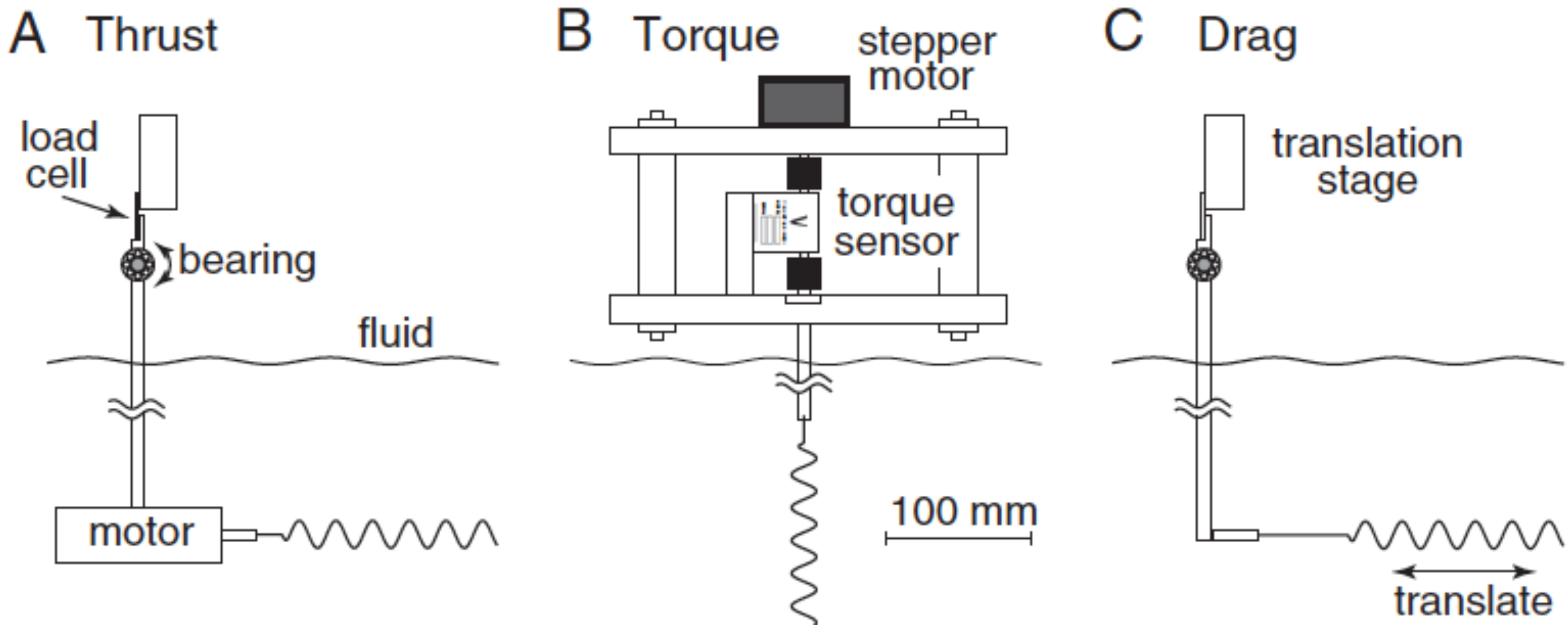
Salmonella: 20nm, Vibrio: 21.5nm, Spirochetes (30.5nm)



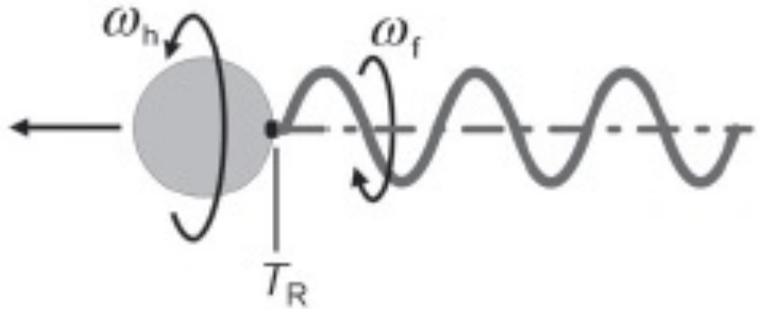
Bacterial Flagellum: Optical Traps



Bacterial Flagellum: Macroscale Models



Bacterial Flagellum: Theoretical Analysis



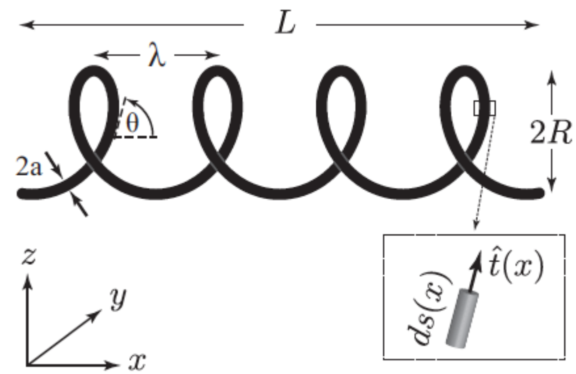


Table 1. Parameters of flagella for several species of bacteria [the filament radius *a* is typically 0.01 μm (20)]

Organism (ref.)	*R*, μm	λ/R	L/λ
Caulobacter crescentus (21)			
Wild type	0.13	8.3	6
Escherichia coli (10)			
CCW	0.195 ± 0.025	11	2.8
Stopped	0.210 ± 0.025	11	2.7
Rhizobium lupini (22)			
Normal	0.250 ± 0.015	5.4	4
Semicoiled	0.385 ± 0.020	2.9	3
Curly	0.135 ± 0.020	9.4	5
Salmonella (23)			
Wild type	0.210 ± 0.005	11	4
Curly mutant	—	—	11
Tumbling mutant	0.145 ± 0.005	7.6	9

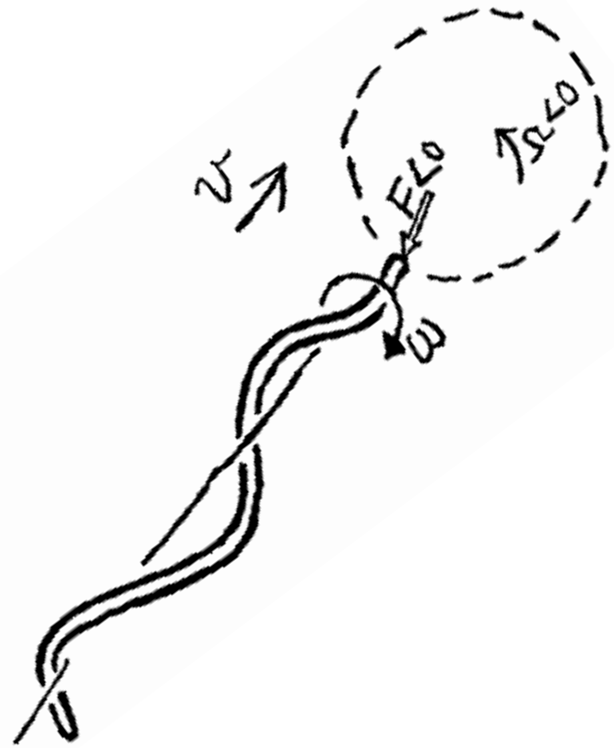
Resistive Force Theory
Slender Body Theory
Regularized Stokeslet Theory

Swimming at low Reynolds Number

$$F = Av + B\omega$$

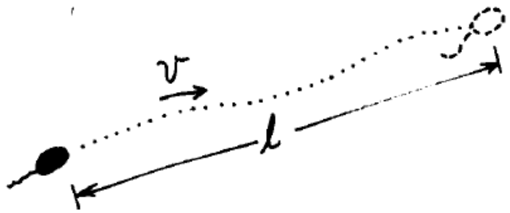
$$N = Cv + D\omega$$

$\begin{pmatrix} A & B \\ C & D \end{pmatrix}$ Propulsion Matrix (Resistance Matrix)



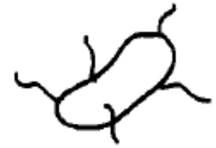
- Thin, perfectly stiff, un-twistable axial wire
- The constants of the propulsion matrix are proportional to the fluid viscosity and depend only on the shape and size of the propeller
- The torque and force on the cell must be equal and opposite to the torque and force on the propeller

Navigation



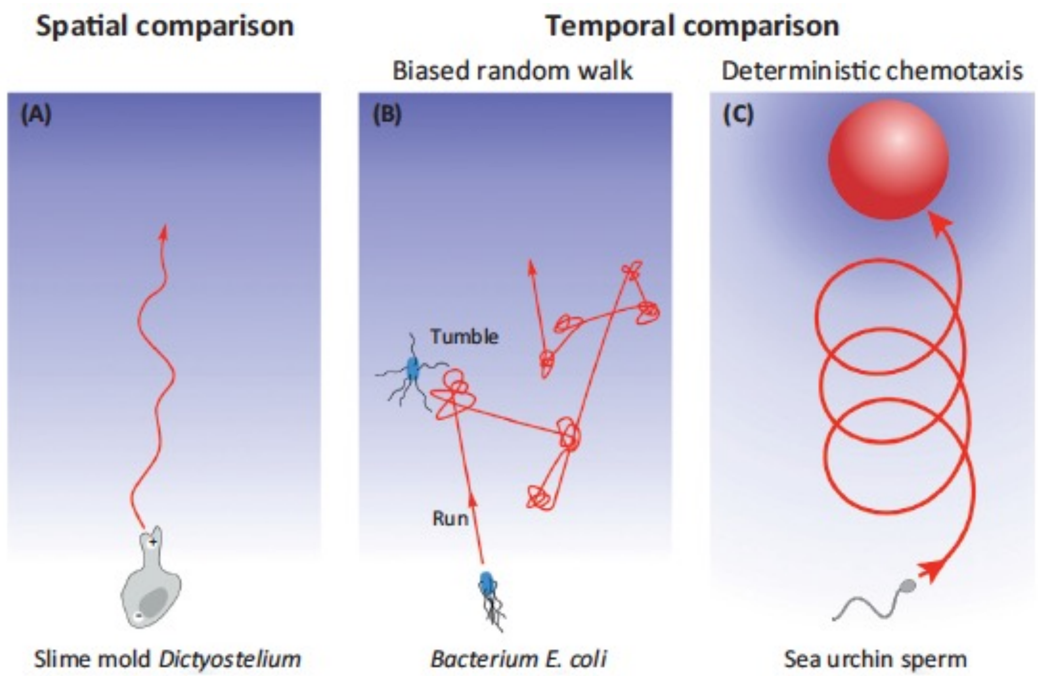
to out-swim diffusion:

$$l \gtrsim D/v \qquad l \gtrsim 30\,\mu$$



$\mathcal{S} \lesssim 10^{-2}$

local stirring accomplishes nothing



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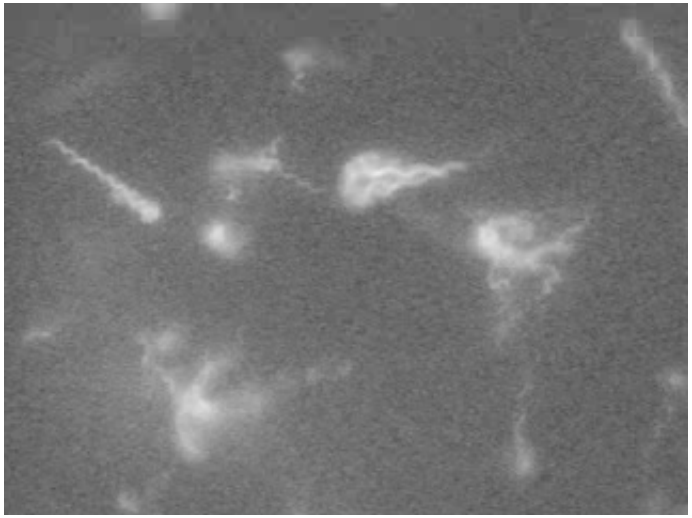
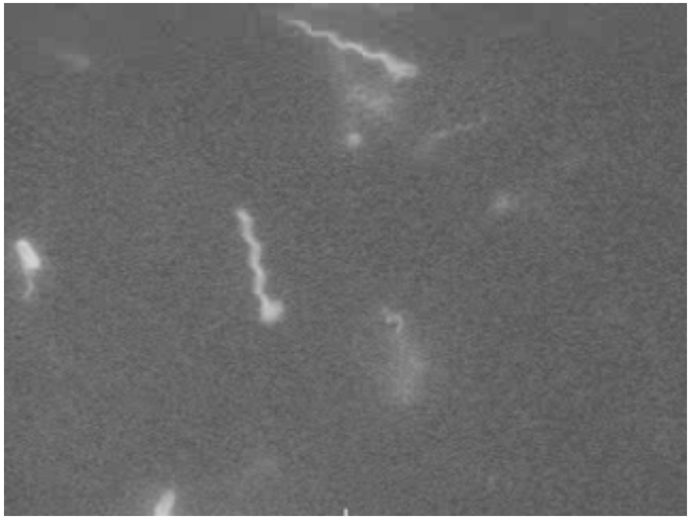
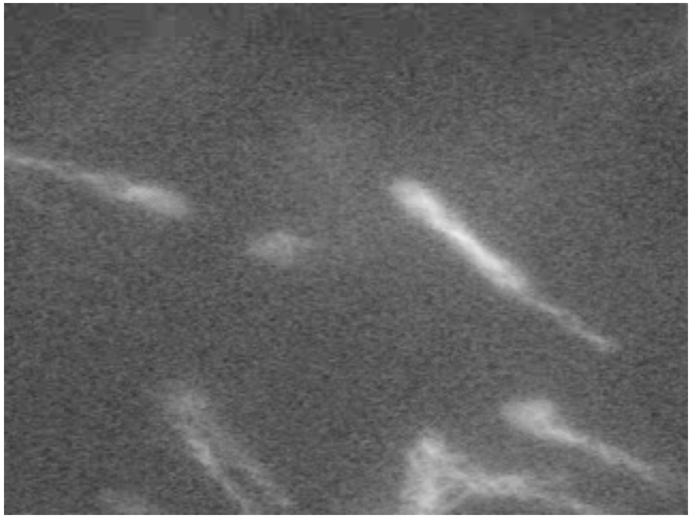
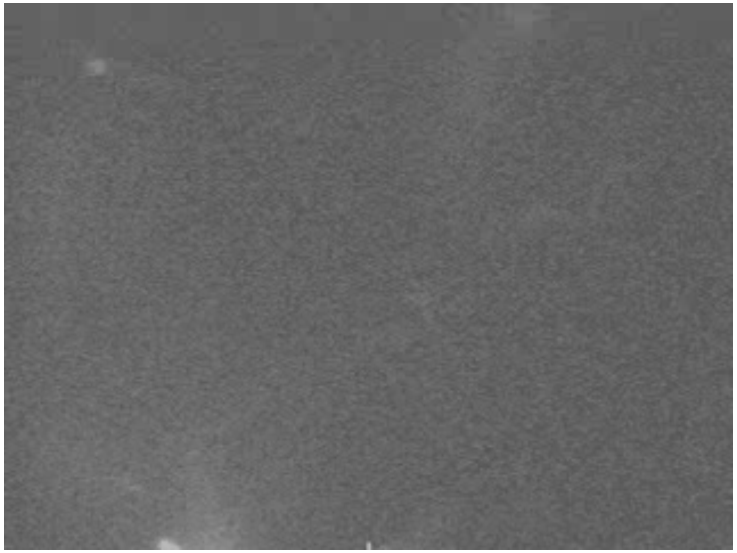
Navigation

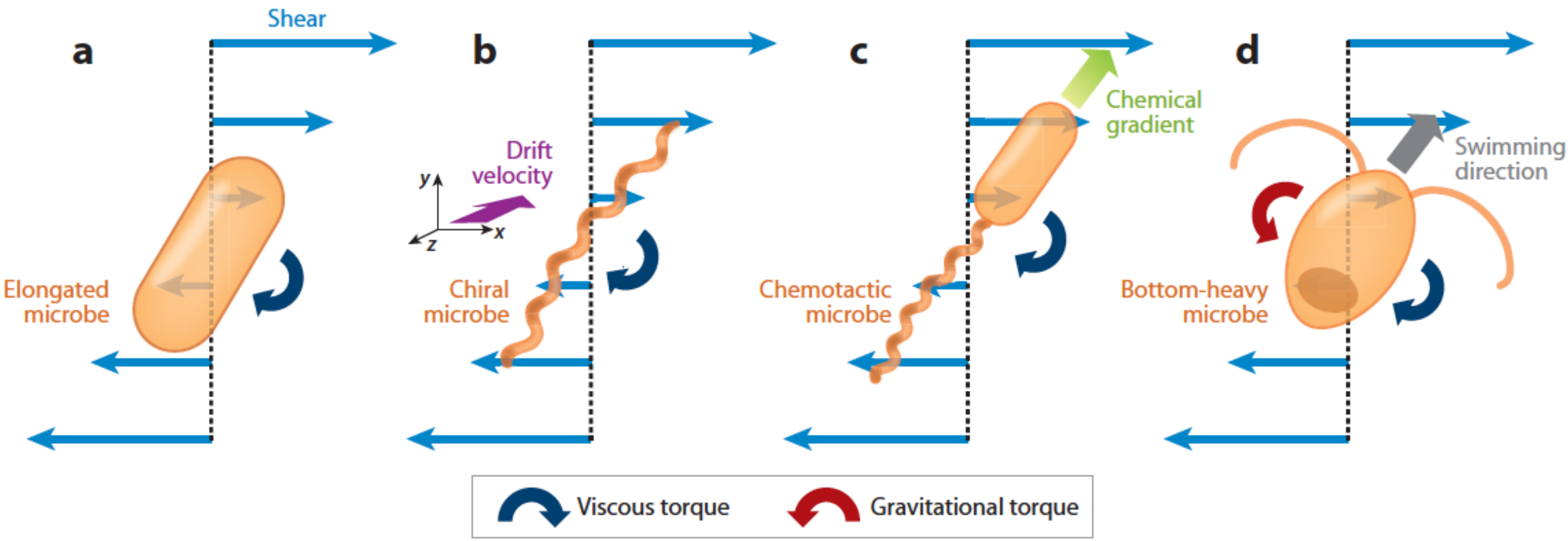






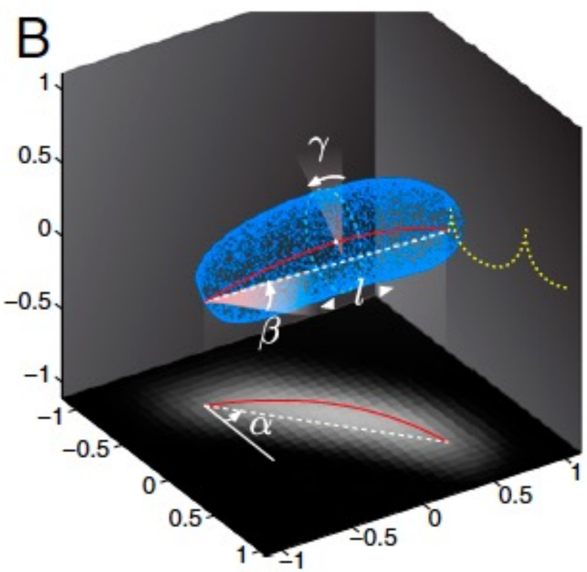


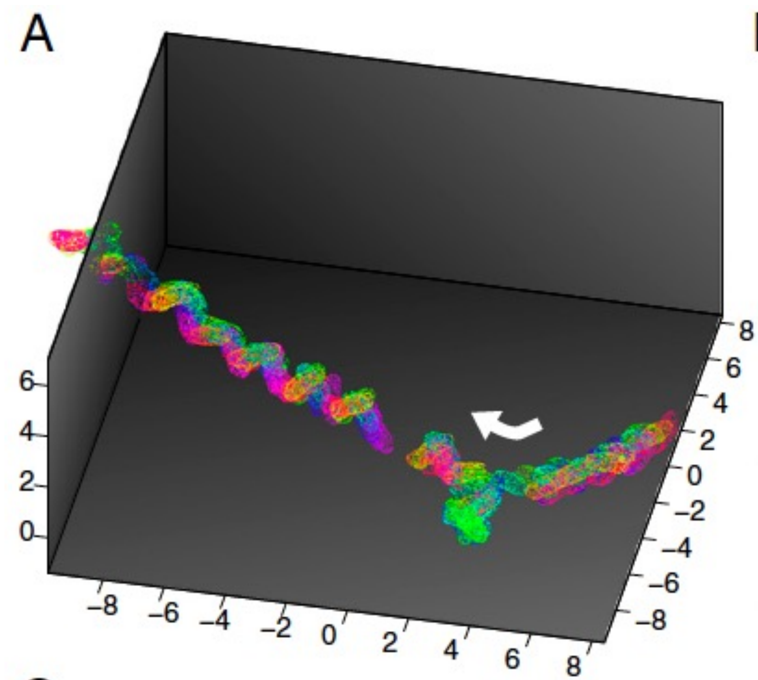
Taxis Behavior



Helical motion of the cell body

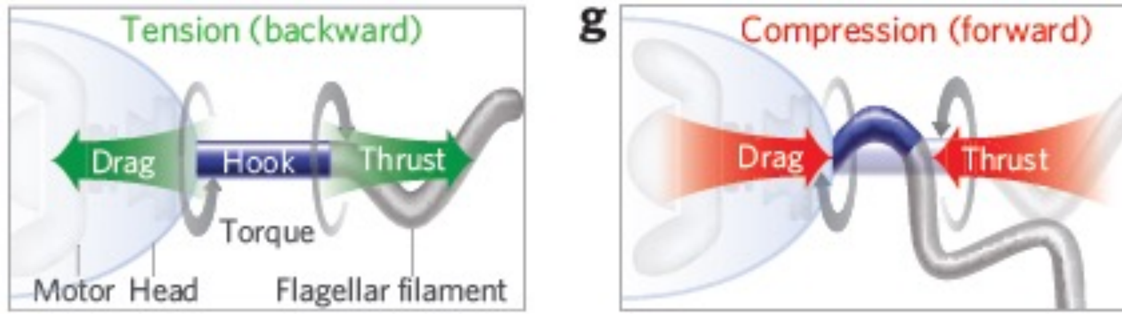
- Uniflagellated *Caulobacter crescentus* display two modes of swimming motility
  - Forward mode: Cell body is tilted wrt the direction of motion (precession due to flexible hook)
  - Reverse mode: Precession is smaller and the cell has a lower motility

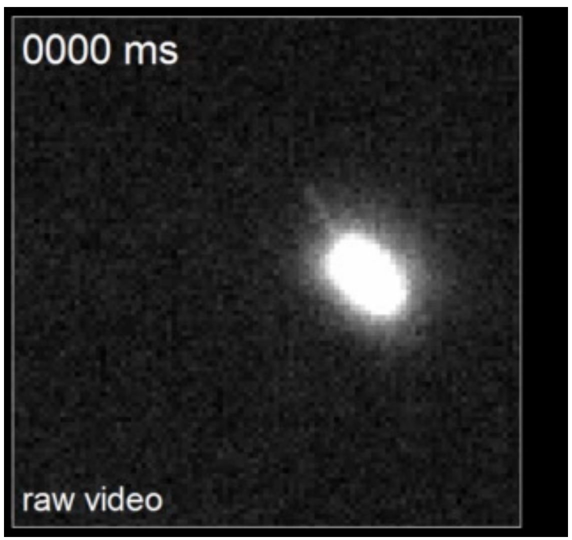


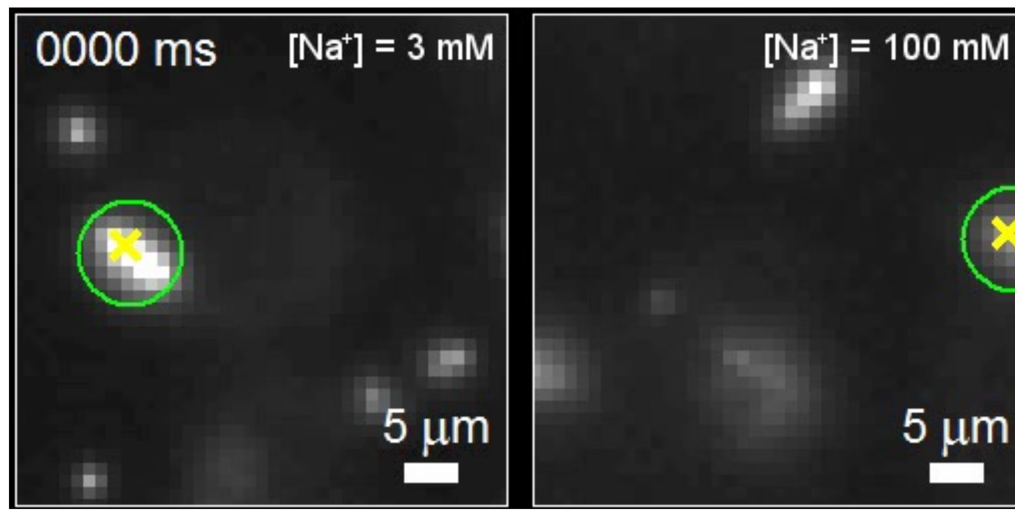


Elastic Instabilities and Changing Direction

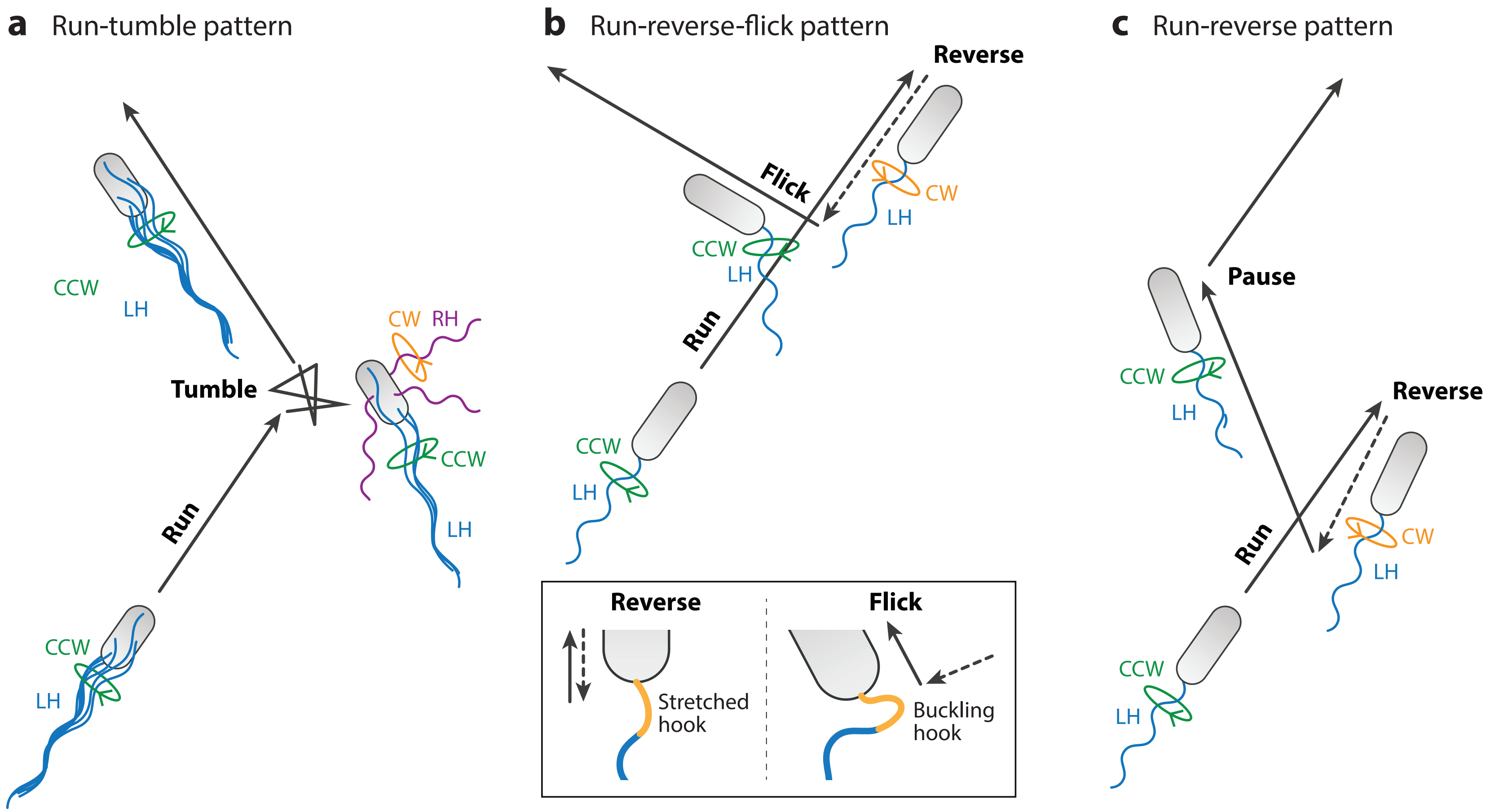
Vibrio alginolyticus





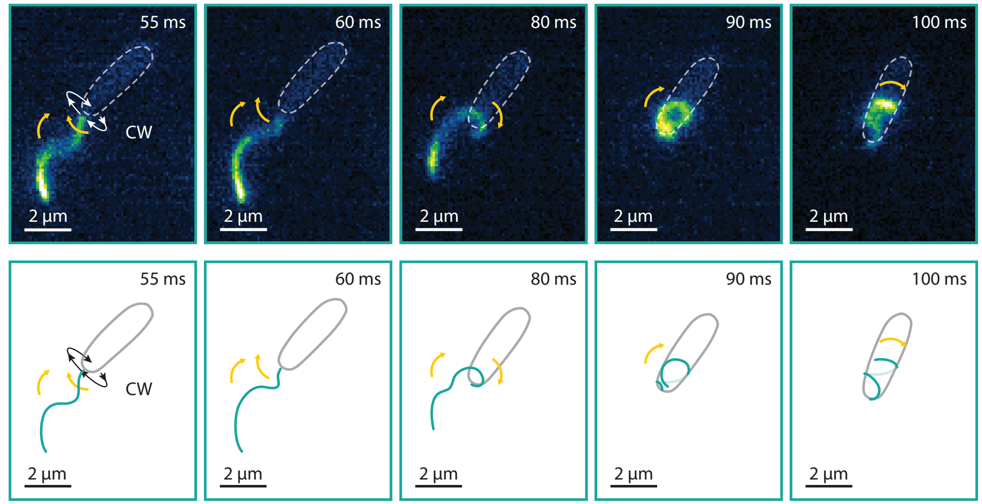


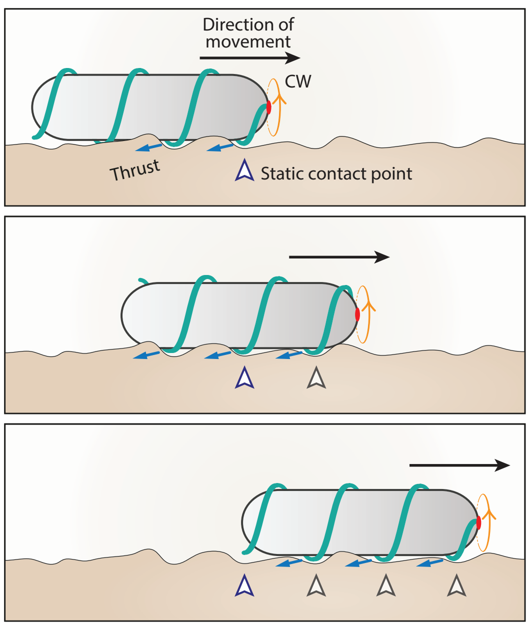
Elastic Instabilities and Changing Direction

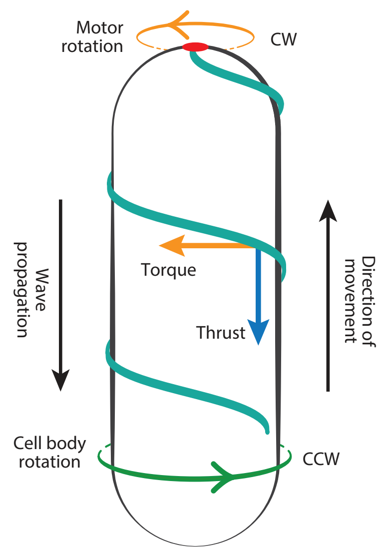


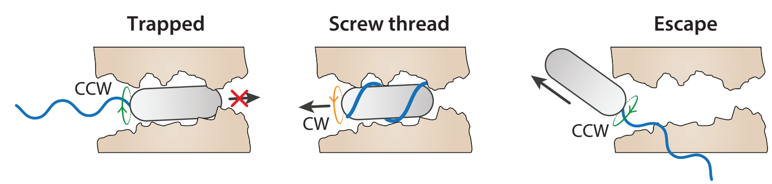
Elastic Instabilities to Escape from Traps

S. putrefaciens

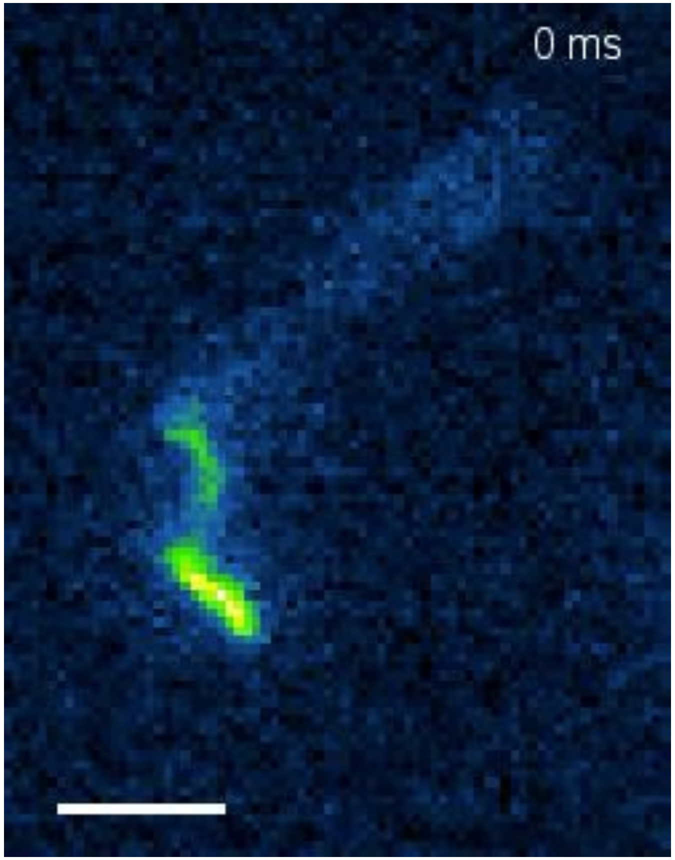


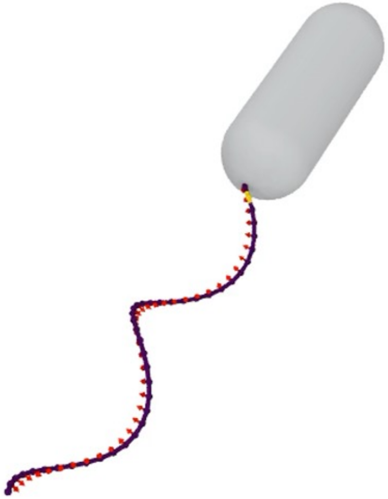




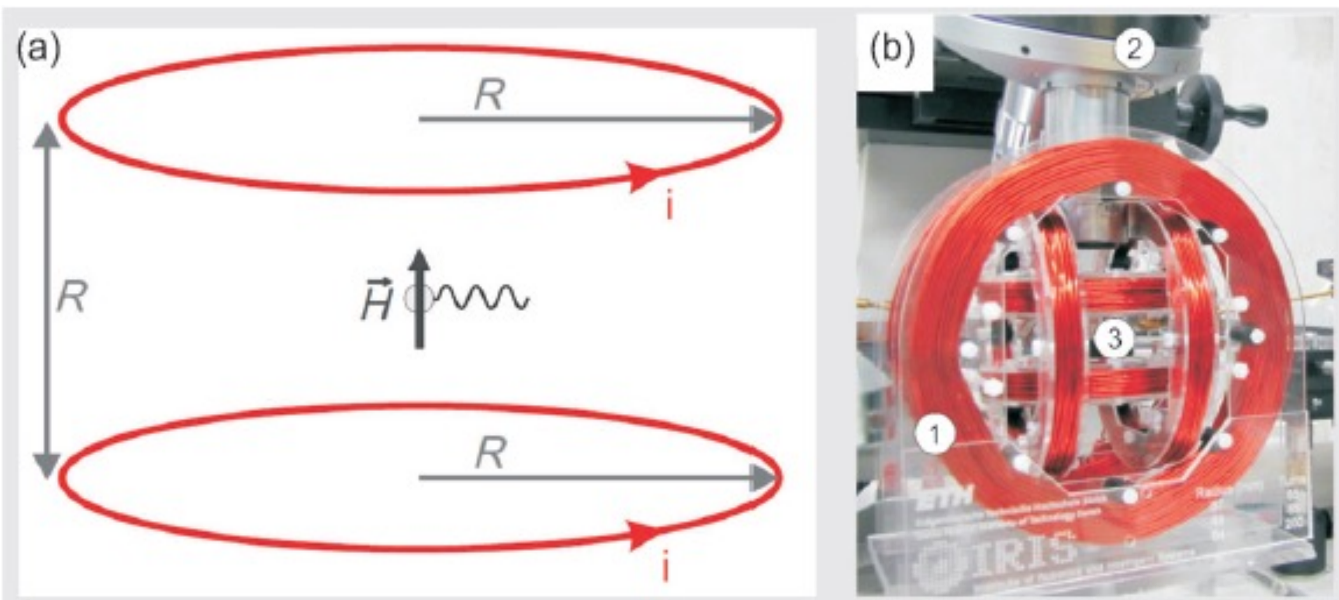


Elastic Instabilities to Escape from Traps

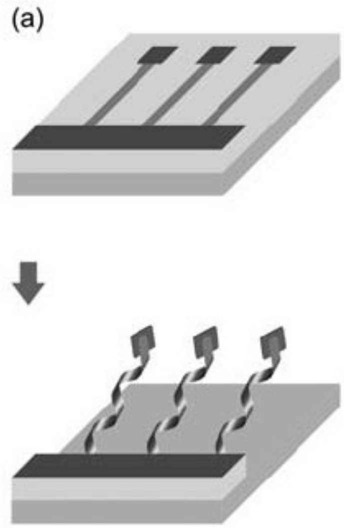


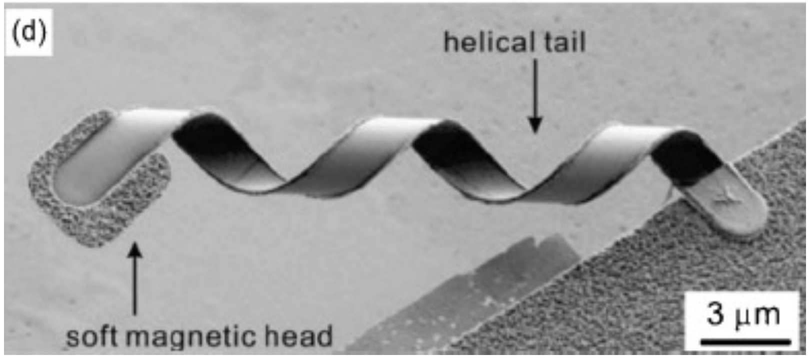


Corkscrew Motion with Artificial Microswimmers



Self-scrolling



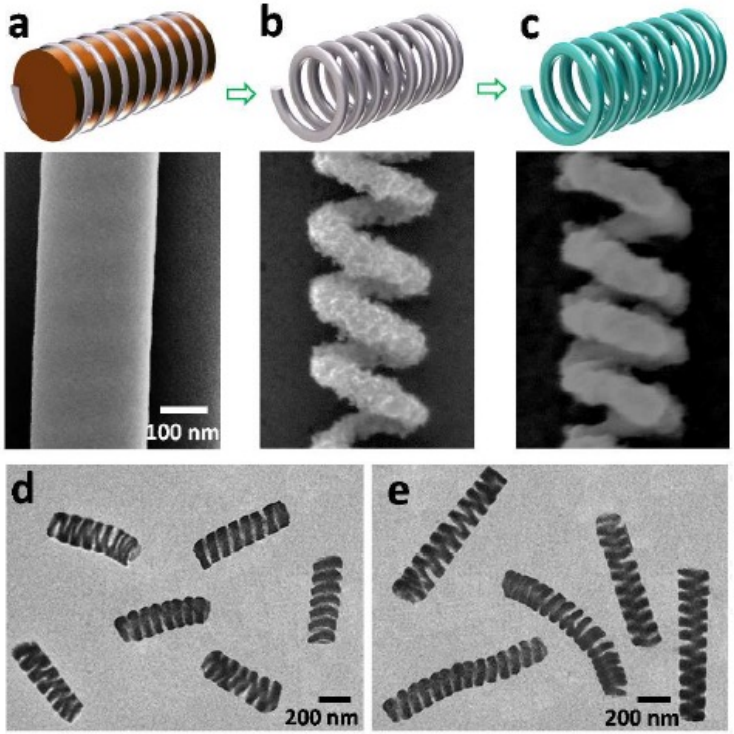


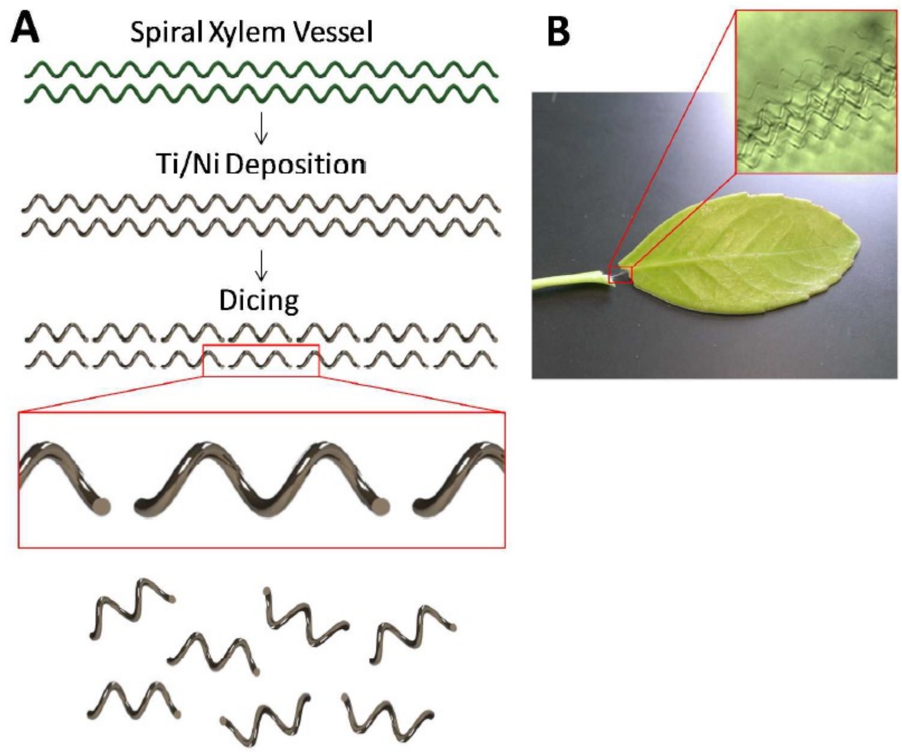
$$R = \frac{(h_1+h_2)\left(8(1+m)^2+(1+mn)(m^2+\frac{1}{mn})\right)}{6\varepsilon(1+m)^2}$$

$$n = E_1/E_2$$

$$m=h_1/h_2$$

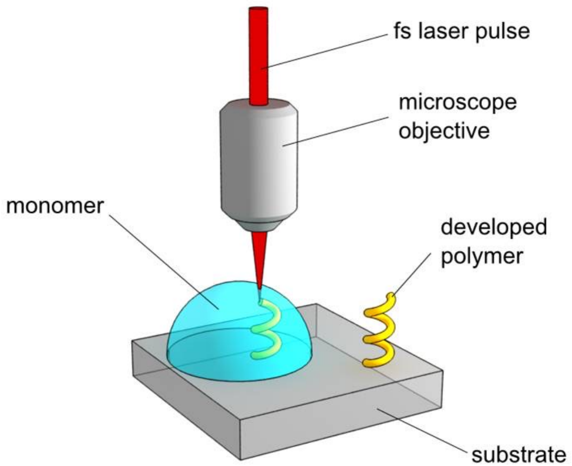
Growing on Seed Material

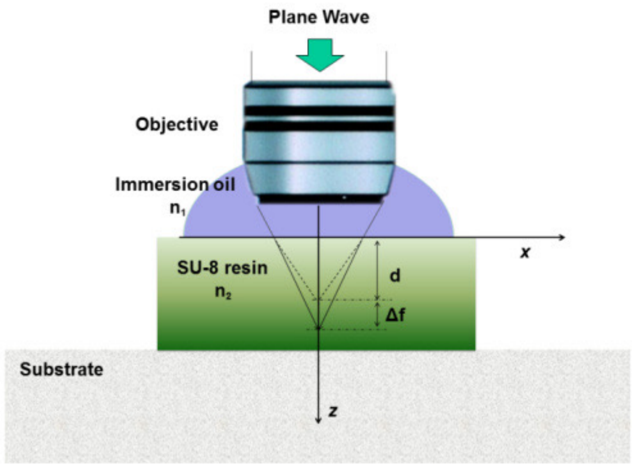


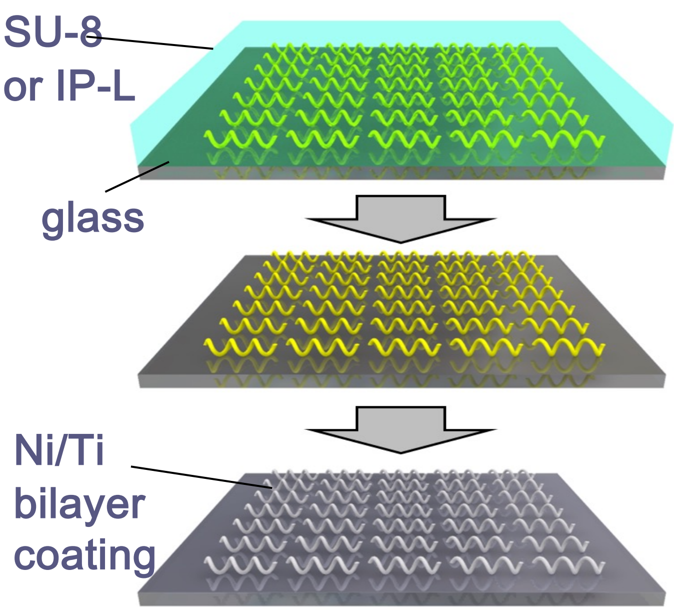


Pd/Cu rods
Cu etching
Ni coating on Pd nanospring

Direct Laser Writing

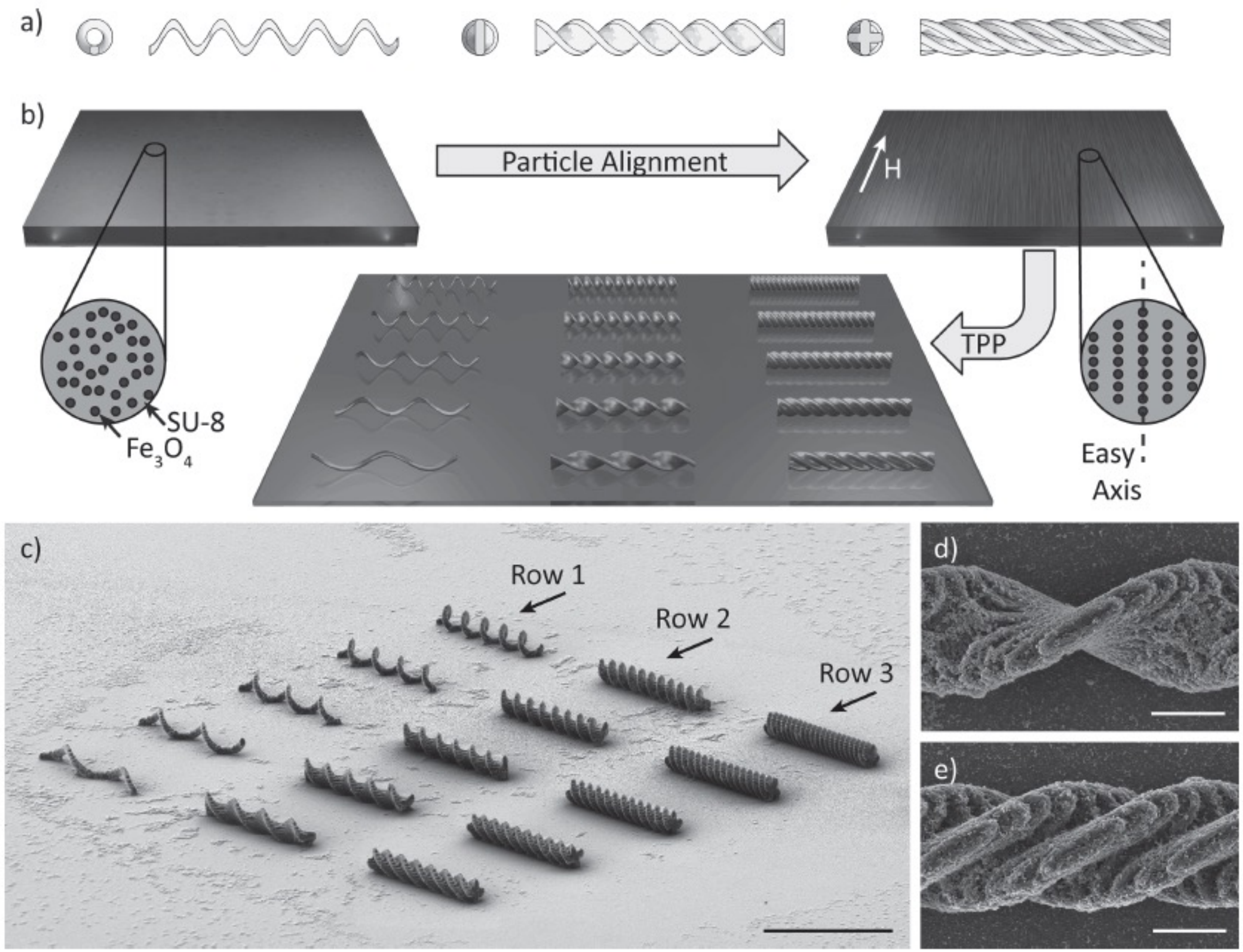






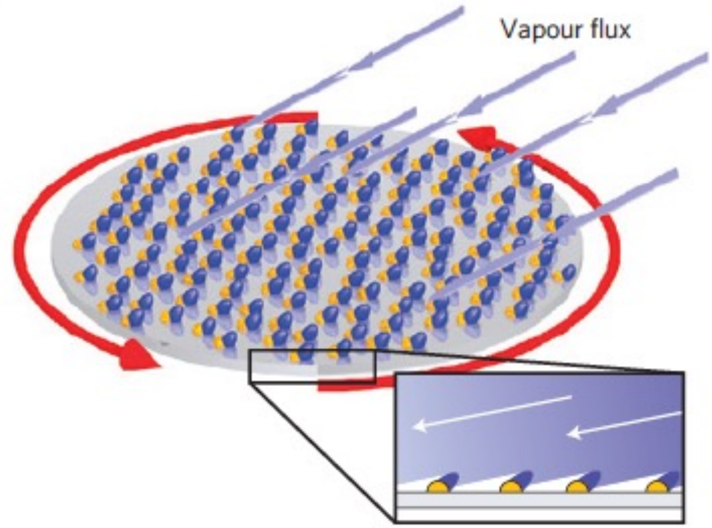
Nanometer scale resolution
Nickel evaporation for magnetization
Titanium evaporation for functionalization

3D Printing of Nanocomposites

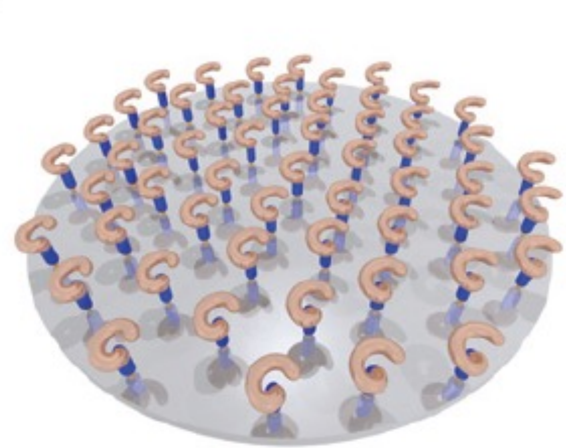


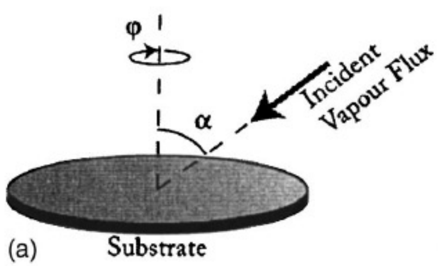
Glancing Angle Deposition

b

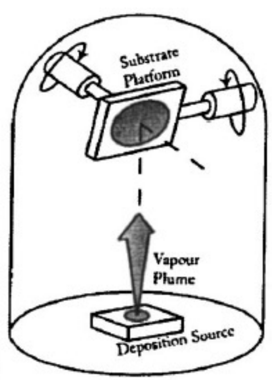


c

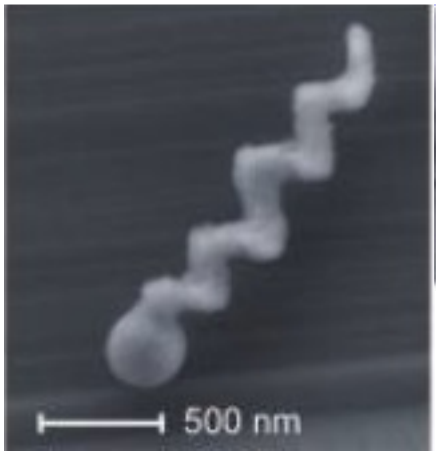


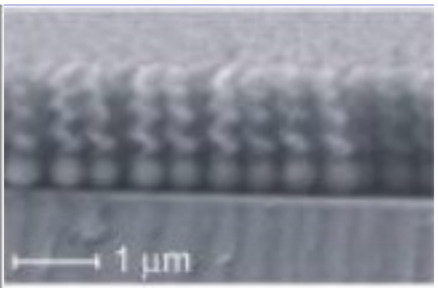


(a)

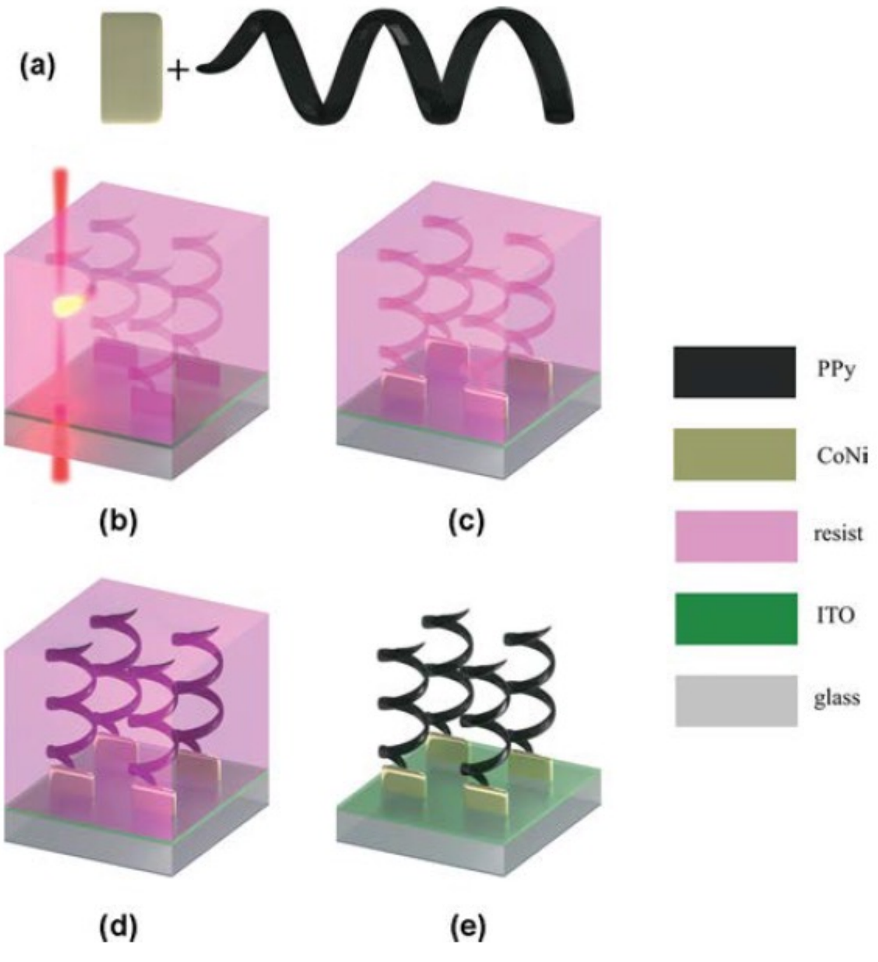


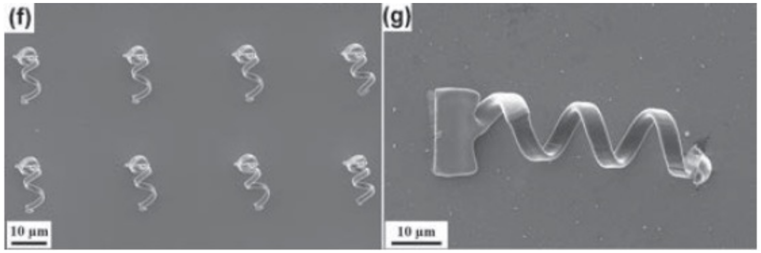
(b)





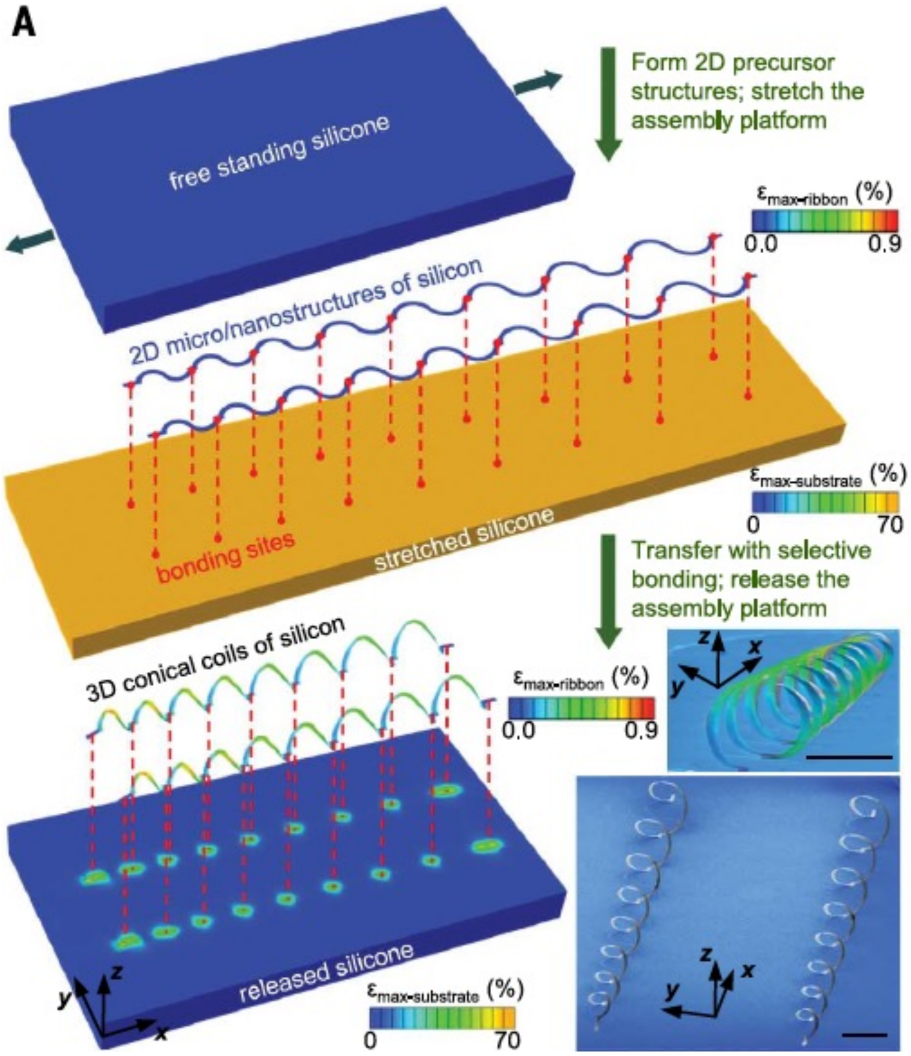
3D Printing and Electrodeposition

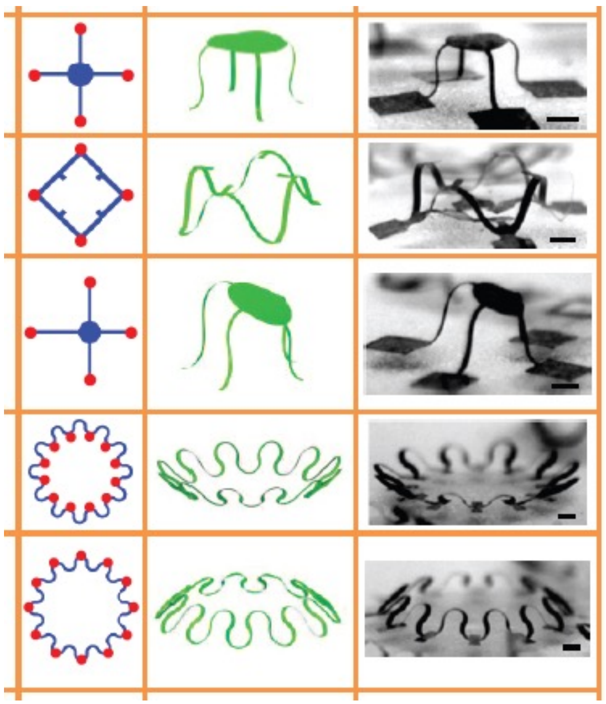




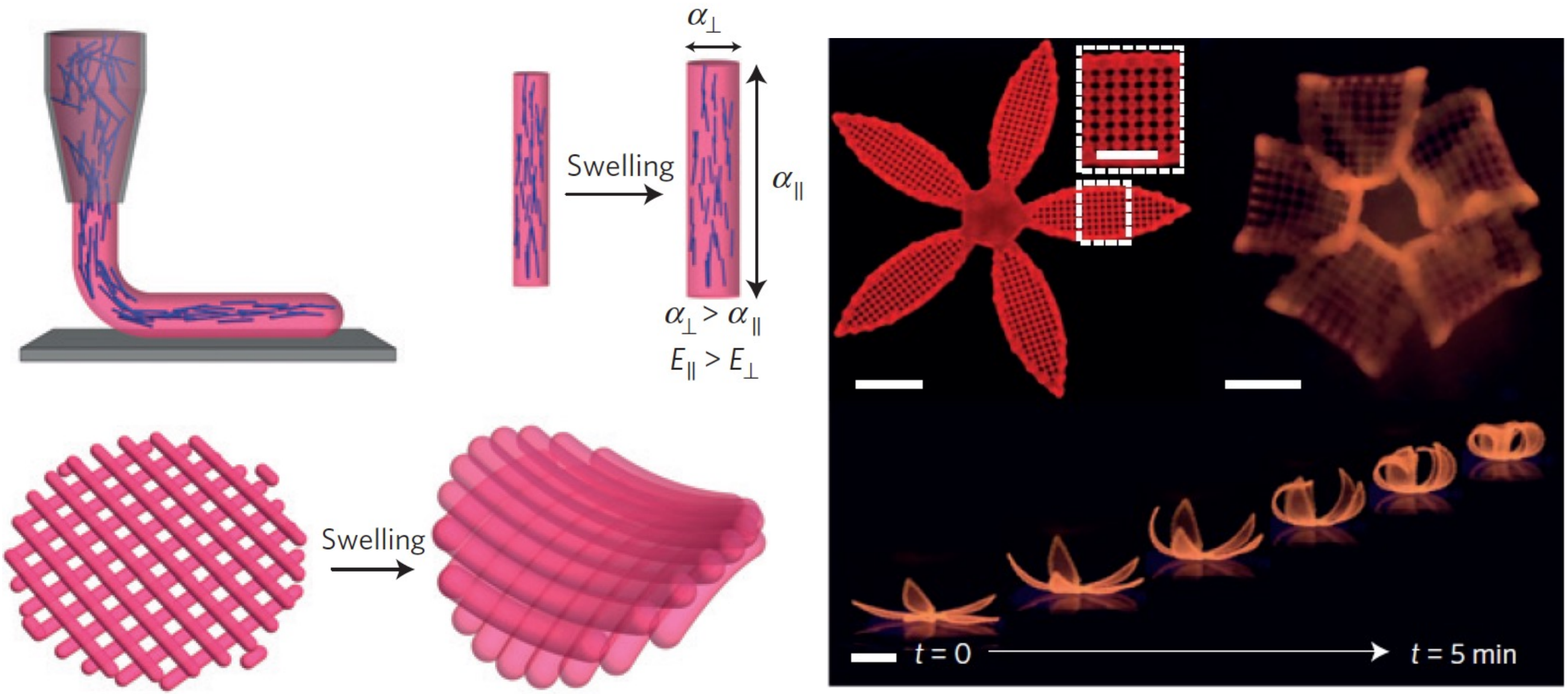
3D photoresist template
Fill with electrodeposition
Magnetic head
Polypyrrole Tail

Compressive Buckling of Silicon



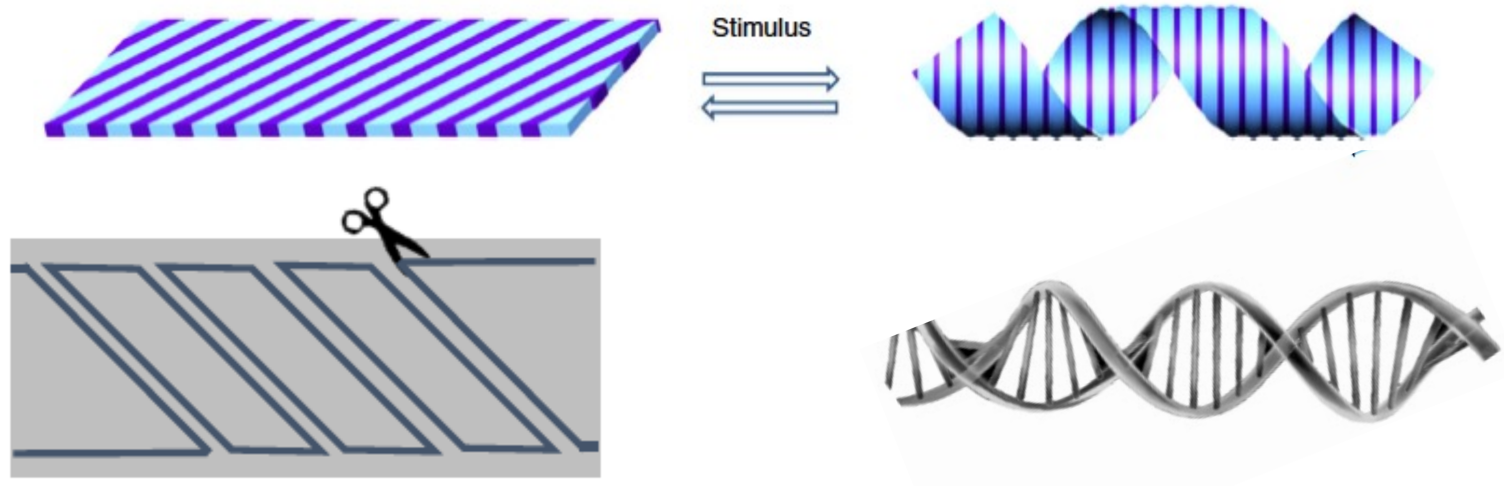


4D Printing



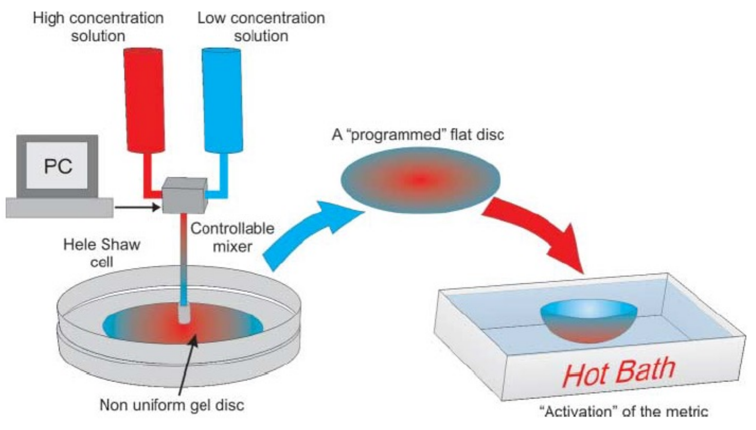
$$\kappa h = \frac{6\Delta\varepsilon(1+m)^2}{3(1+m)^2 + (1+mn)(m^2 + \frac{1}{mn})}$$

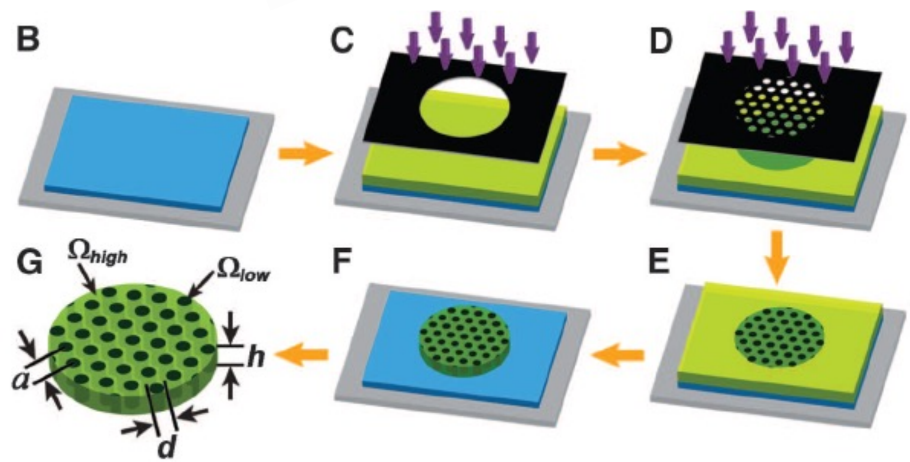
Origami and Kirigami with Hydrogels



Kim etal, *Science*, 2012

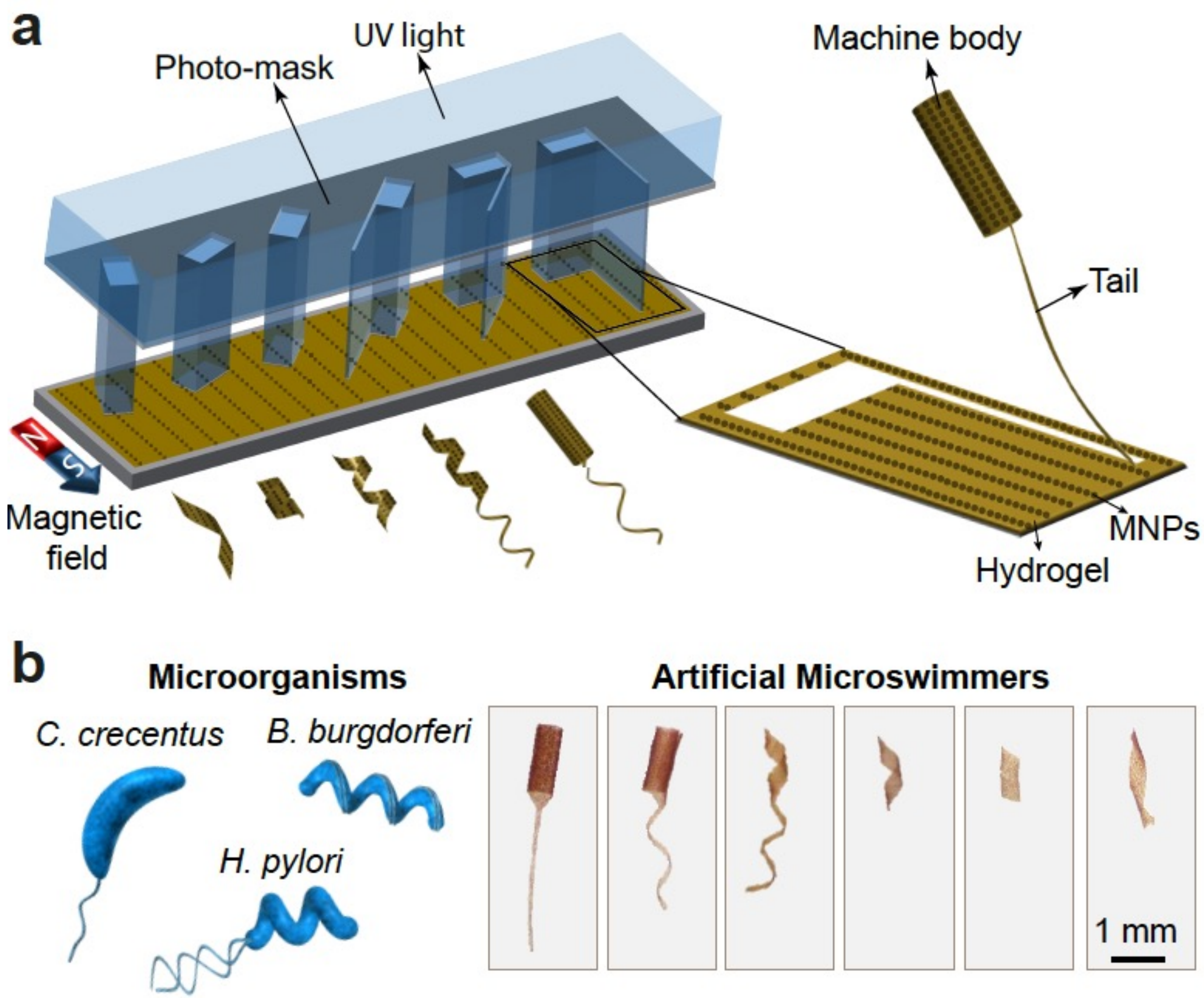
Klein etal, *Science*, 2007



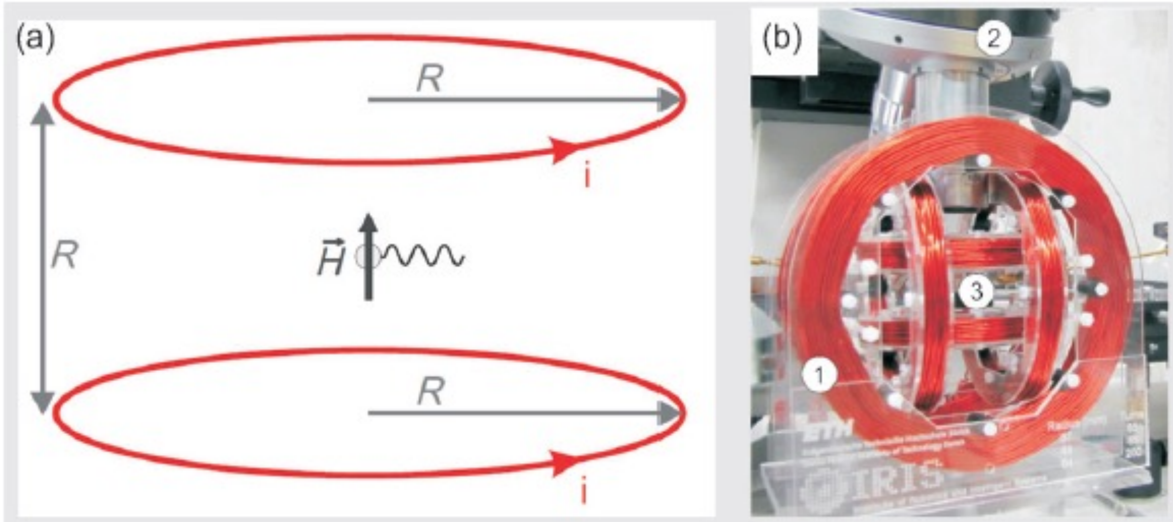


Programmable self-folding

- Differential Swelling via Particle Gradients

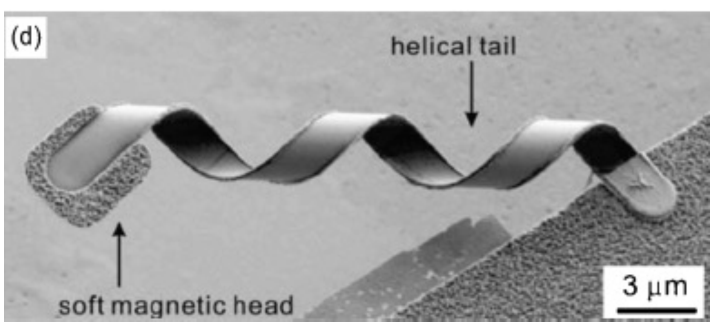


Artificial Microswimmers



Zhang, **APL**, 2010

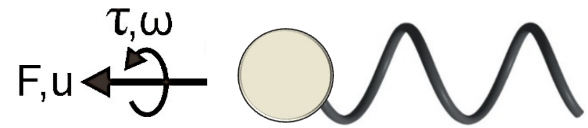
$$\mathbf{T} = v\mathbf{M} \times \mathbf{B}$$



Propulsion Matrix

- Linear relationship between force F, torque τ, velocity u and rotational speed ω

$$\begin{pmatrix} F \\ \tau \end{pmatrix} = \begin{pmatrix} a & b \\ b & c \end{pmatrix} \cdot \begin{pmatrix} u \\ \omega \end{pmatrix}$$



- Measuring the parameter of the propulsion matrix
  - Gravity compensation
  - Free-fall
  - Horizontal swimming

$$\begin{pmatrix} a & b \\ b & c \end{pmatrix} = \begin{pmatrix} 1.5 \cdot 10^{-7} & 1.6 \cdot 10^{-14} \\ 1.6 \cdot 10^{-14} & 1.5 \cdot 10^{-19} \end{pmatrix}$$

Propulsion Matrix

- **Experiment 1**: Vertical balancing (u = 0)
  - ABF in vertical position
  - Propulsive force equalizes the external forces (gravity & buoyancy)

$$F_{ext} = -F_{grav} + F_{buoy}$$

$$F = a \cdot u + b \cdot \omega \quad \text{(i)}$$

$$\tau = b \cdot u + c \cdot \omega \quad \text{(ii)}$$

$$b = \frac{F_{ext}}{\omega}$$

- Experiment: tune ω until ABF does not move out of focus anymore
  - F_{ext} = - $5.1 \cdot 10^{-13}$ N, ω = 31 rad/s

b = $-1.6 \cdot 10^{-13}$ N·s

Propulsion Matrix

- **Experiment 2**: Vertical free-fall (τ = 0)
  - ABF in vertical position
  - Free-fall velocity

$$F = a \cdot u + b \cdot \omega \quad \text{(i)}$$

$$\tau = b \cdot u + c \cdot \omega \quad \text{(ii)}$$

$$c = -\frac{b \cdot u}{\omega}$$

- Experiment: switch off actuation and record speed
  - ω = - 0.28 rad/s

c = 2.3·10^{-19} N·s·m

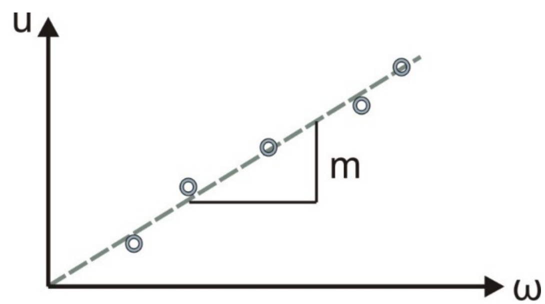
Propulsion Matrix

- **Experiment 3**: Horizontal swimming (F = 0)
  - ABF in horizontal position

$$F = a \cdot u + b \cdot \omega \quad \text{(i)}$$

$$\tau = b \cdot u + c \cdot \omega \quad \text{(ii)}$$

$$u = -\underbrace{\frac{b}{a}}_{=:\, m}\omega$$

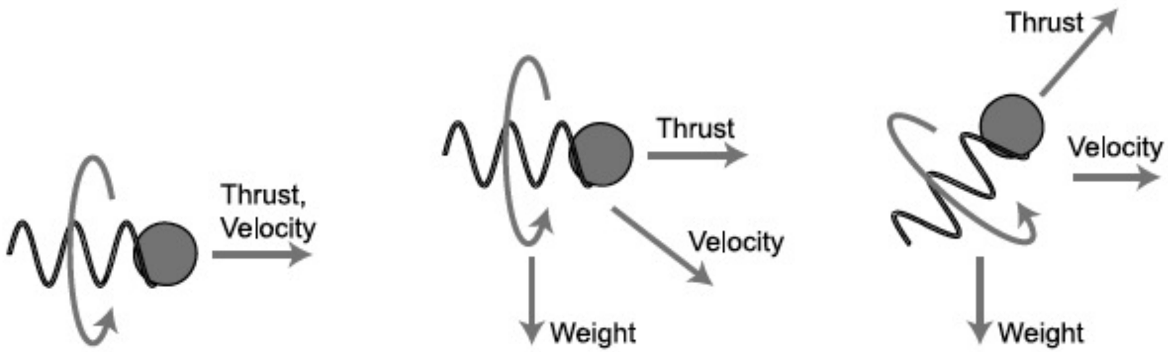


- Experiment: drive ABF at different frequencies and record velocities
  - Extract slope m of the linear ω-u relationship
  - m = 1.1·10-7 m/rad

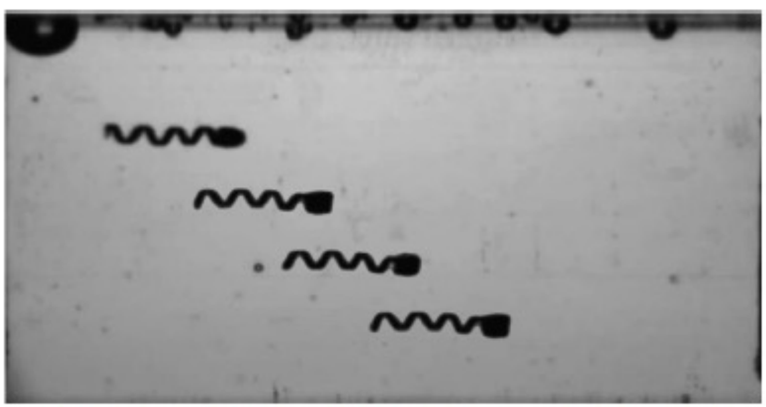
a = 1.5·10^{-7} N·s/m

Gravity Compensation

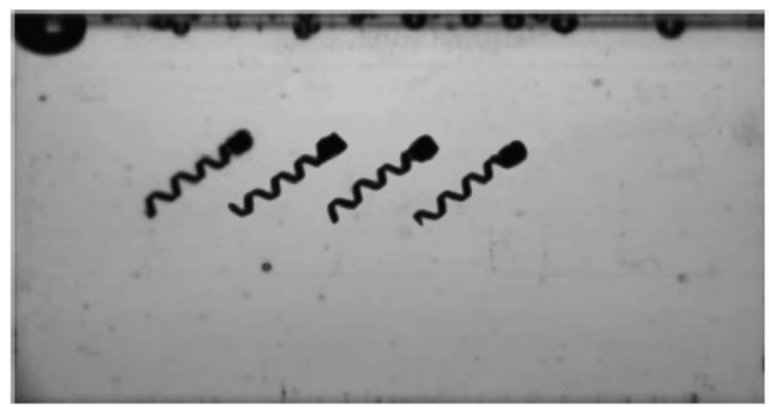
- Density depends on material choice
- Moving up against gravity



(a) Neutrally buoyant swimmer (b) Heavy swimmer (c) Gravity compensation

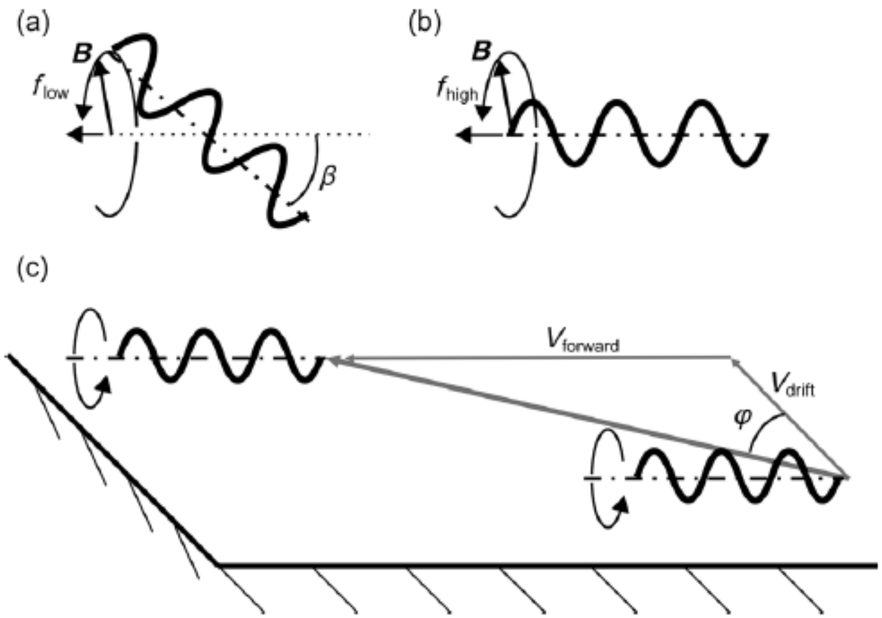


(d) Experiment without gravity compensation



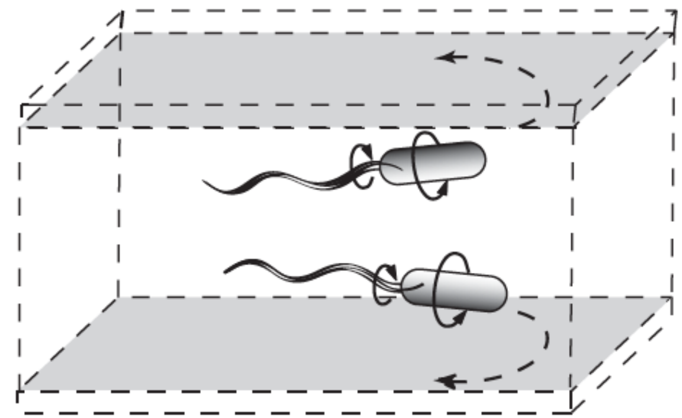
(e) Experiment with gravity compensation

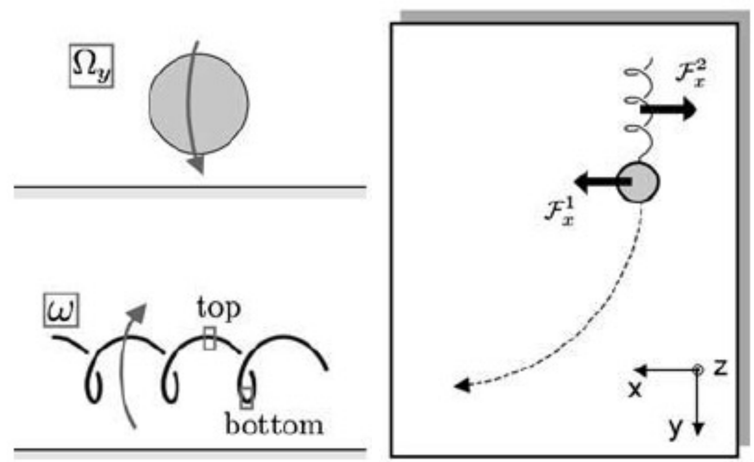
Wobbling Motion and Drift



Huang et.al., *APL*, 2014

Bacteria swim in circles near planar surfaces

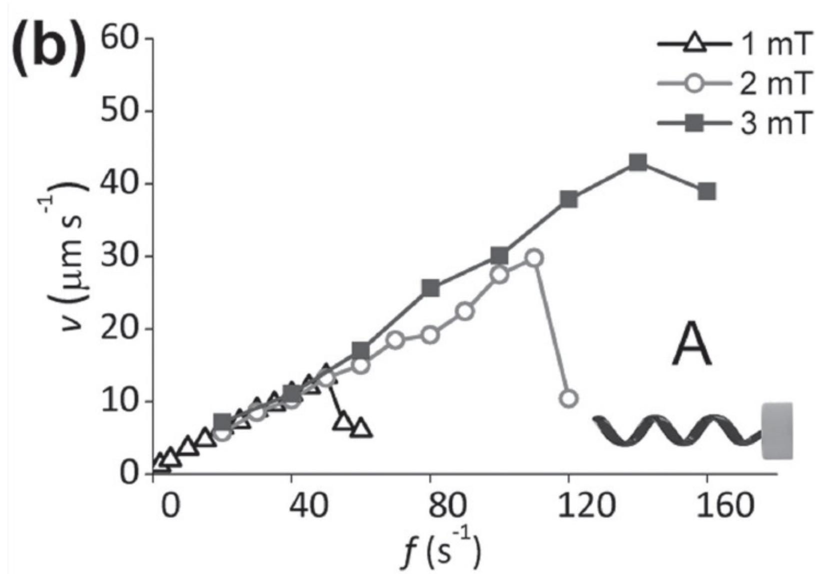




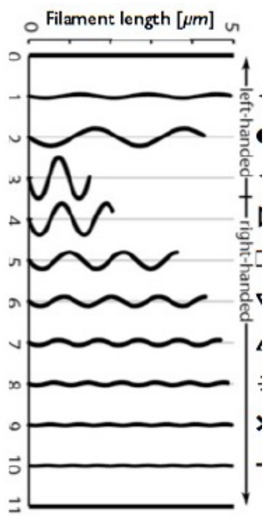
Step-out Frequency

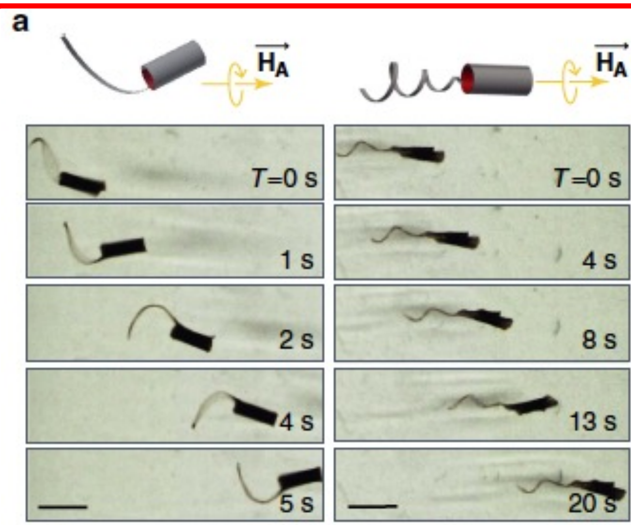
- When the applied magnetic field rotates sufficiently slowly, the robots synchronously rotate with the field
- There exists a rotation frequency above which the applied magnetic torque is not strong enough to keep the robot synchronized with the field
  - Step-out frequency
- Step-out frequency depends on
  - Robot magnetization
  - Friction
  - Field strength
- Robot's velocity rapidly declines

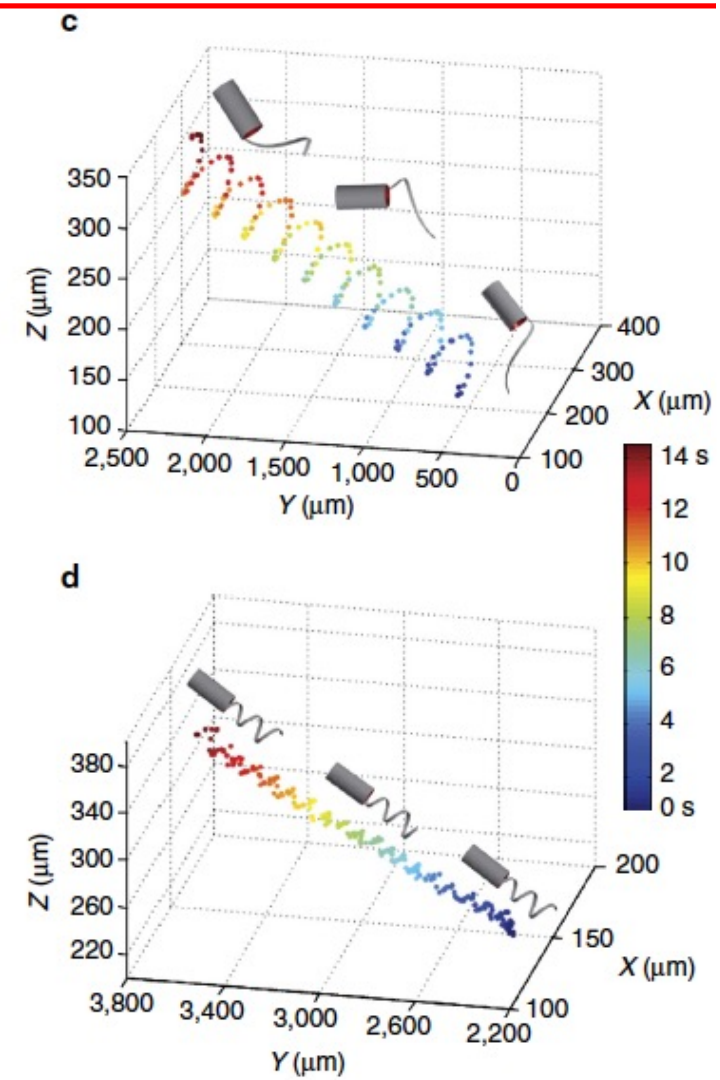
when operated above step out frequency



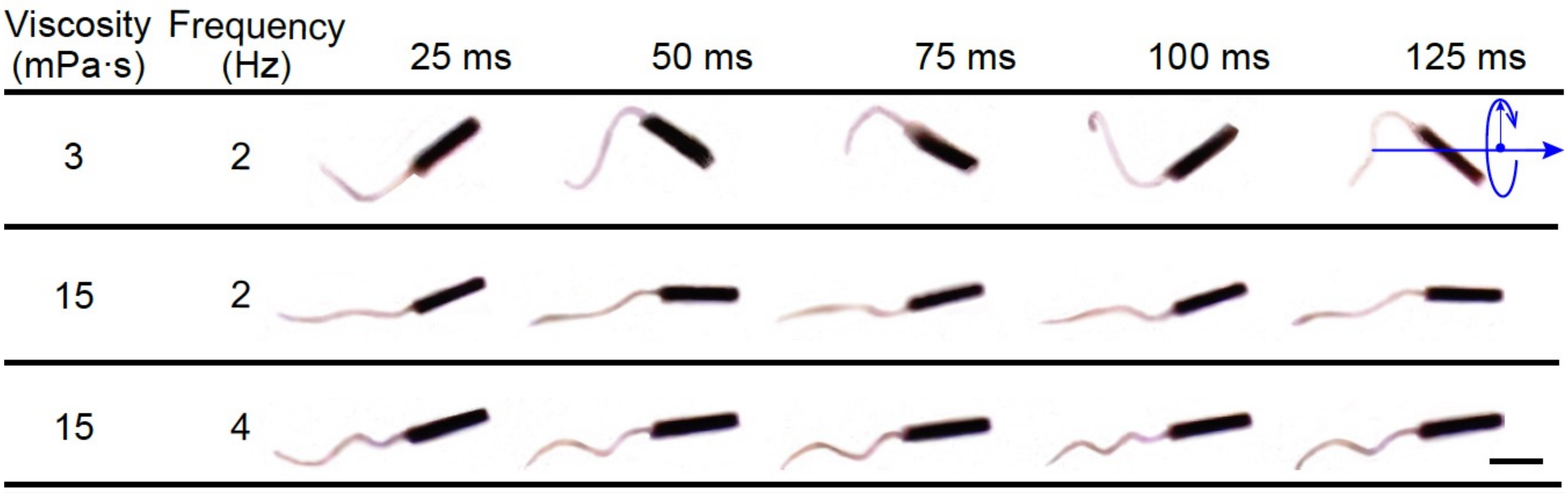
The role of tail geometry







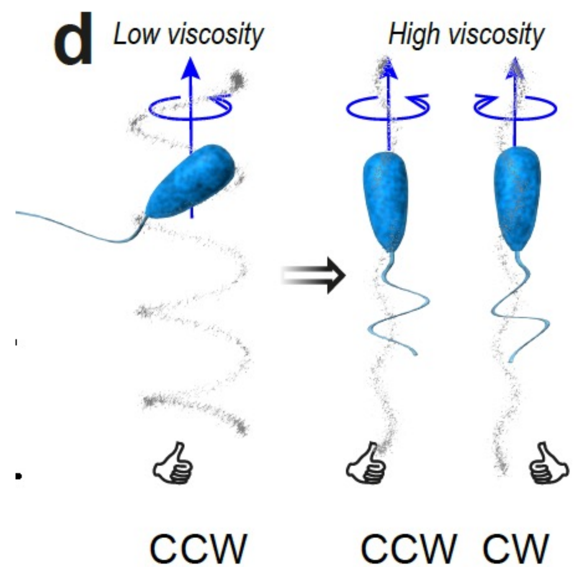
Elastohydrodynamic Coupling



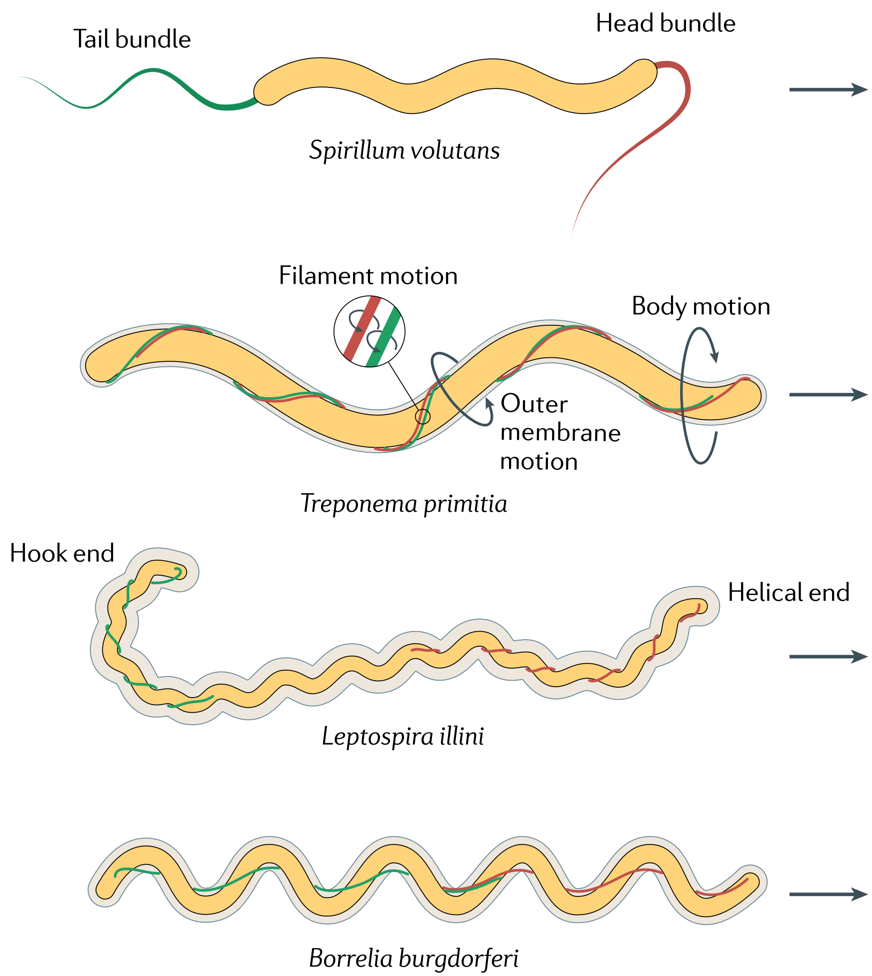
Sperm Number

$$S_p = L/\left(\frac{\kappa}{\zeta_\perp \omega}\right)^{1/4}$$

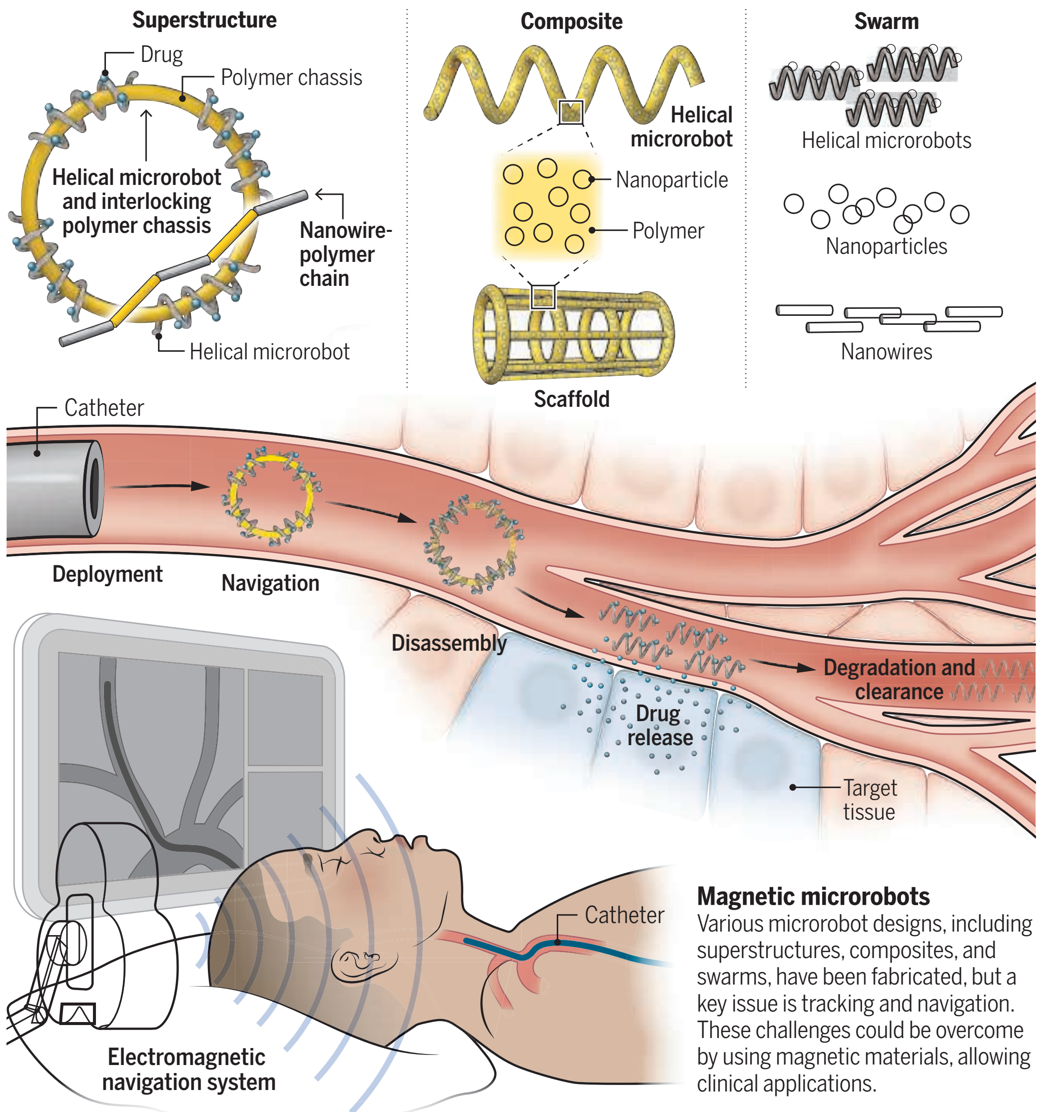
Bending rigidity vs viscous drag



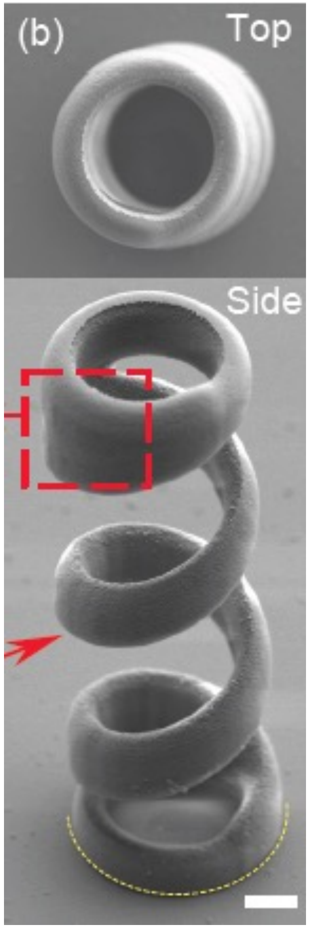
Swimming with helical bodies

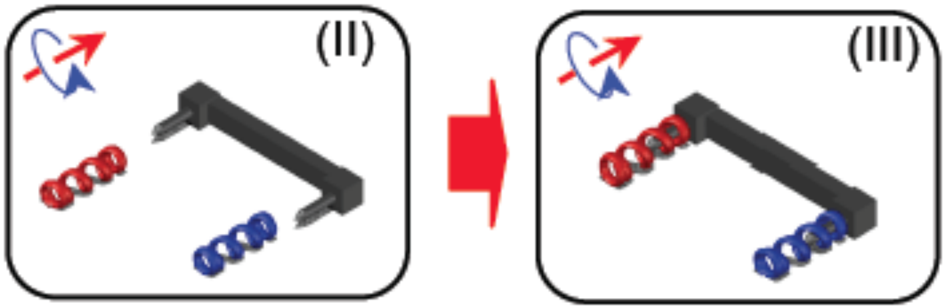


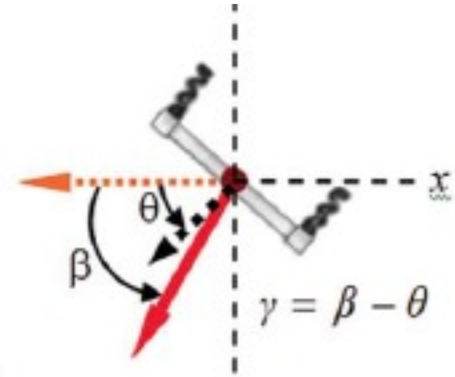
Delivering drugs with microrobots

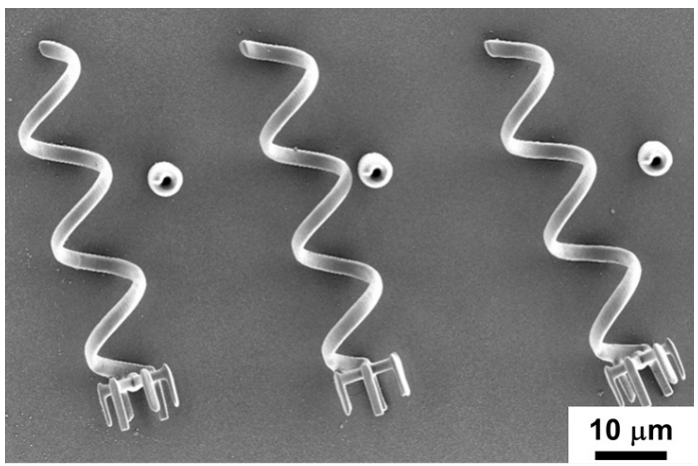


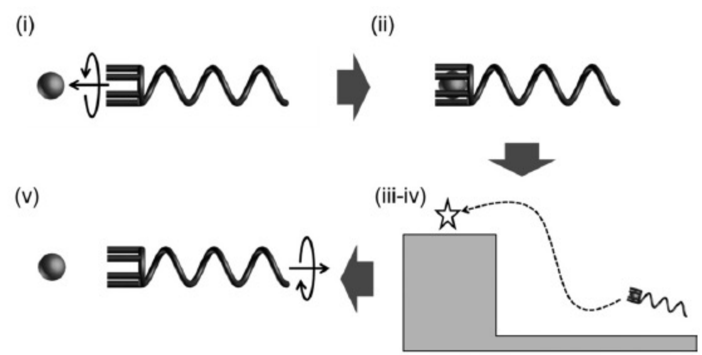
Microtransporter (contact mode)





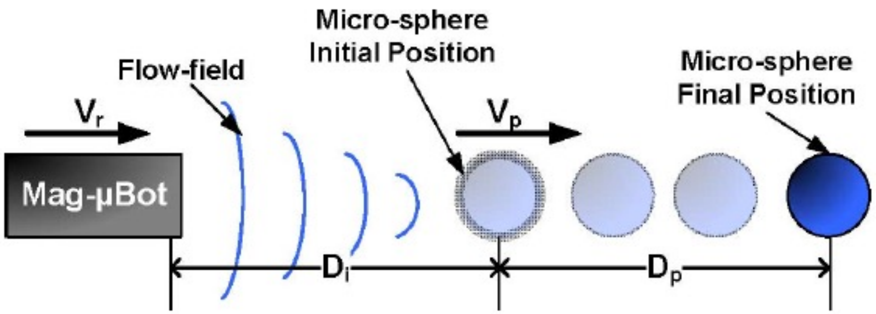


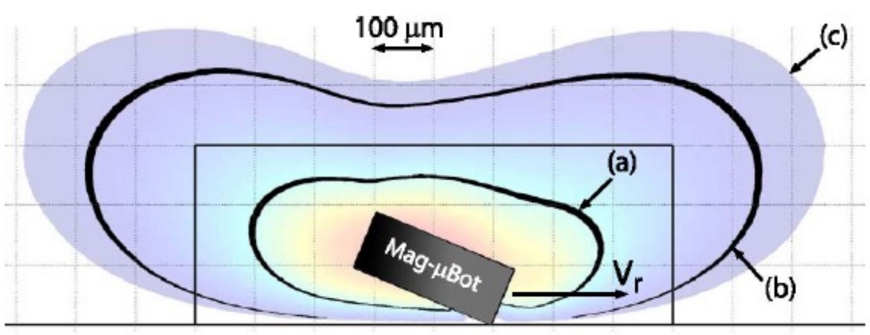


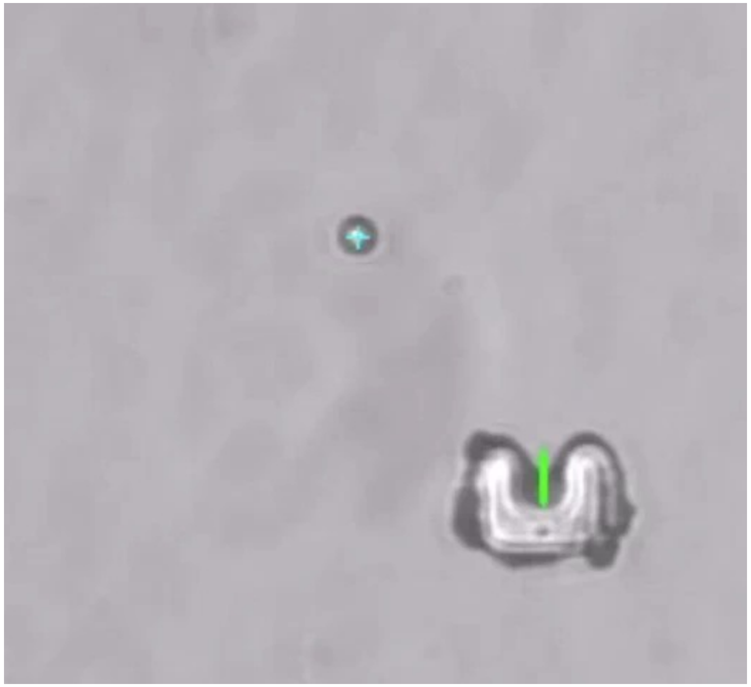


Microtransporter (non-contact mode)

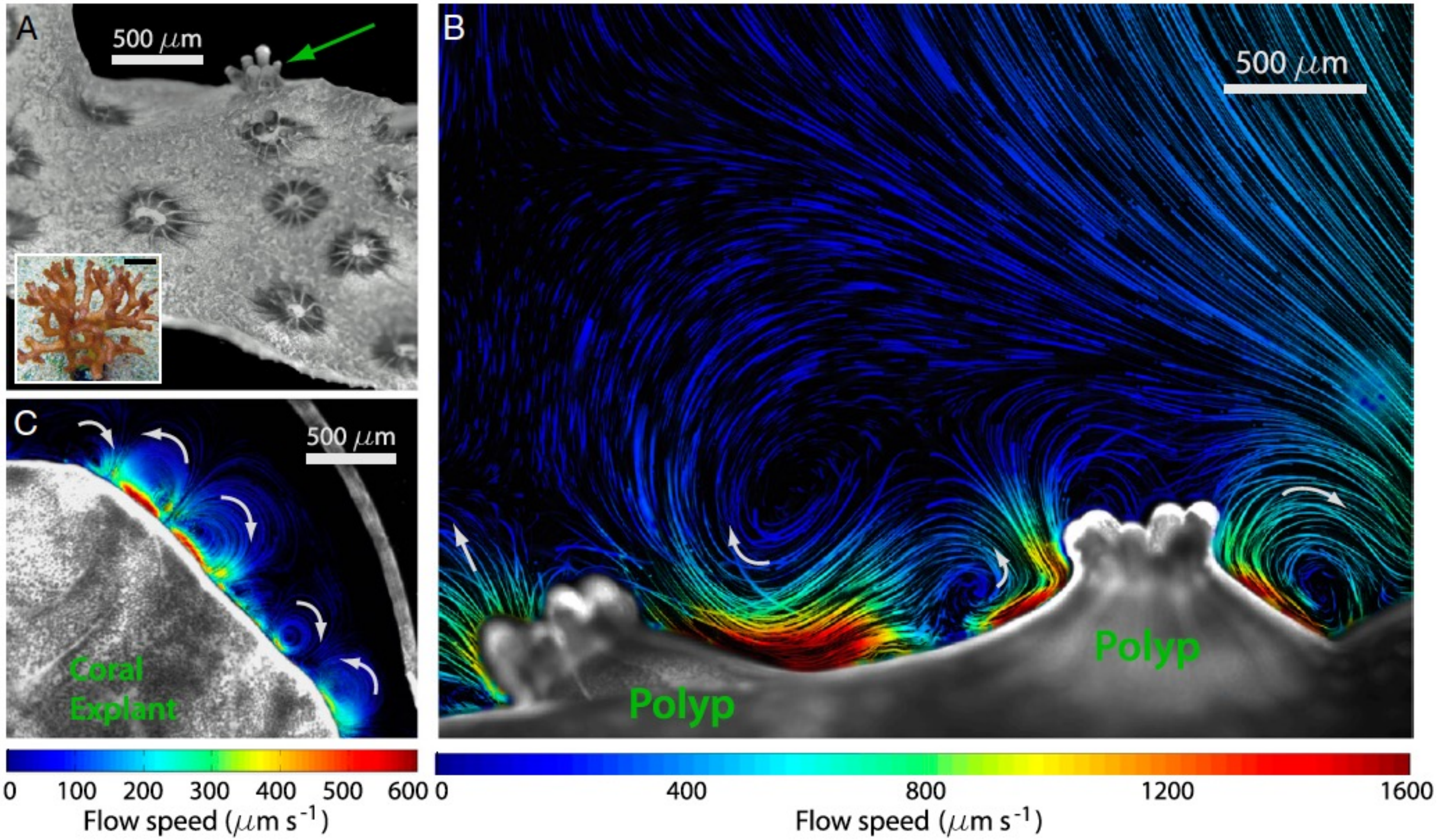
- Compartmentalization (robot and payload)
- Fluidic coupling



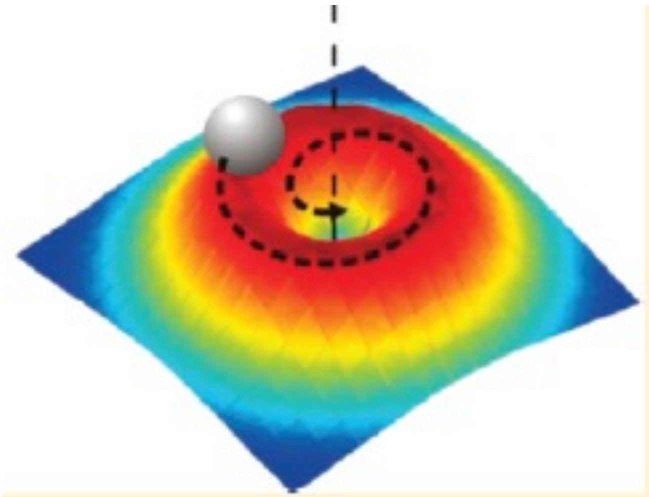




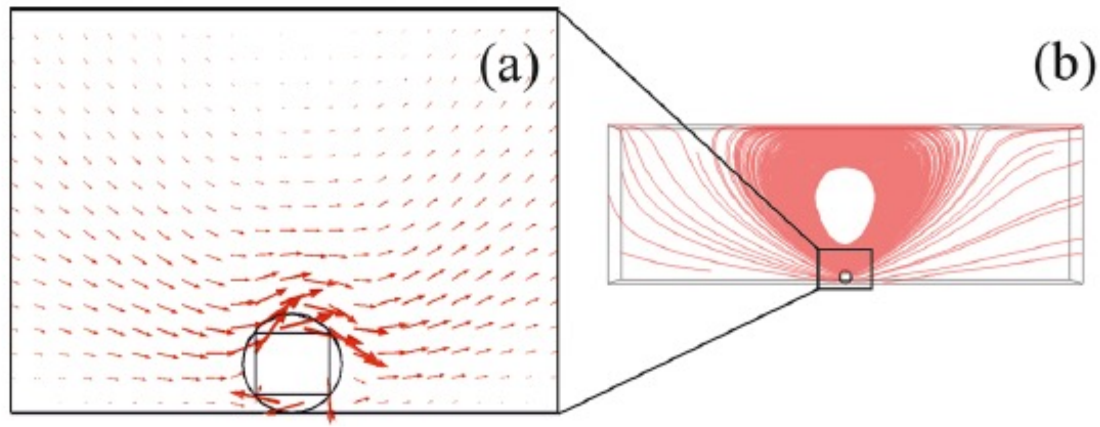
Active Transport at Low Reynolds Number

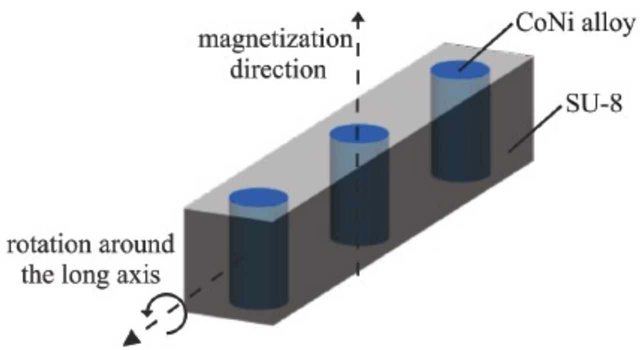


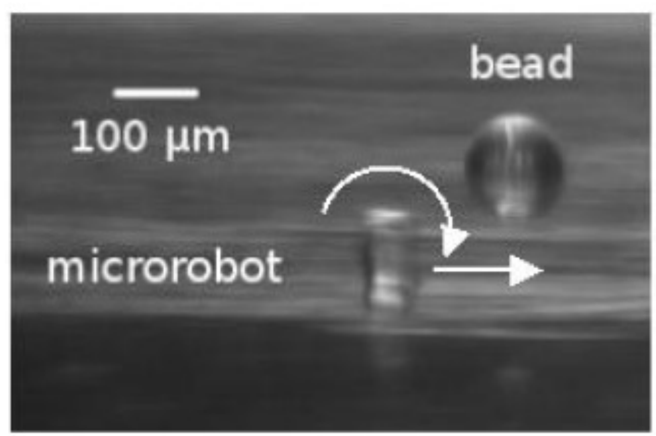
Generating Mobile Microvortices

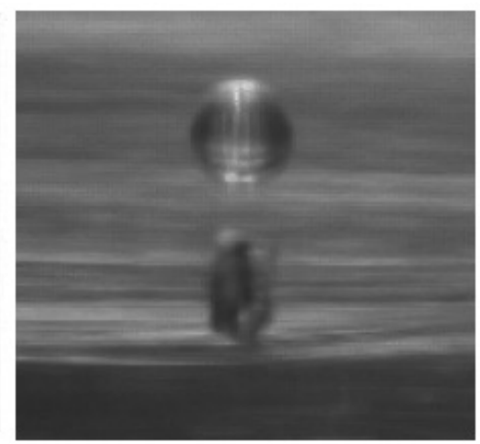


$$\nabla p = \eta \nabla^2 \boldsymbol{U} + \boldsymbol{f},$$

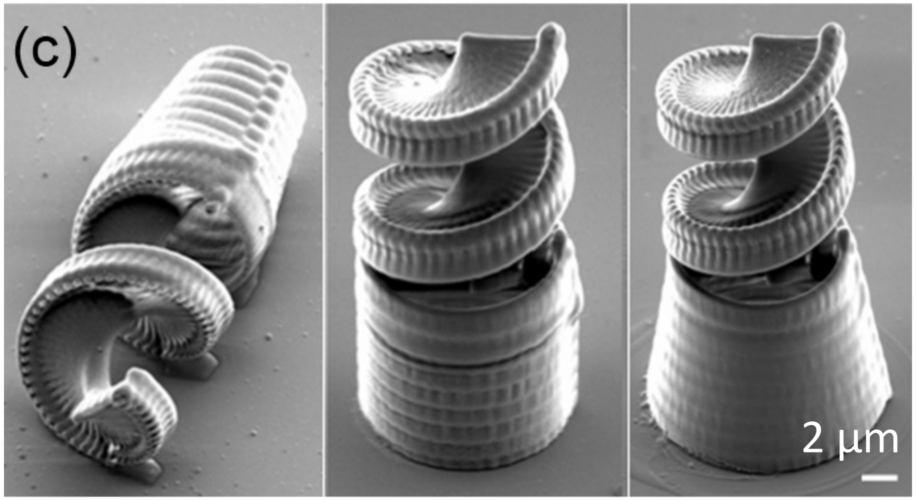


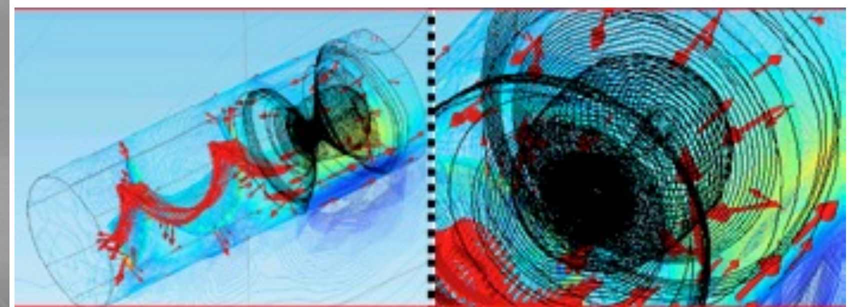


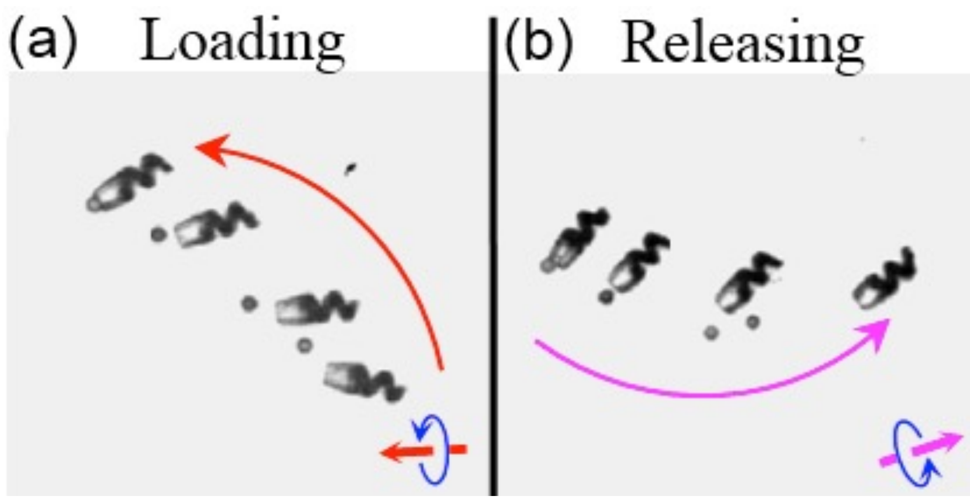




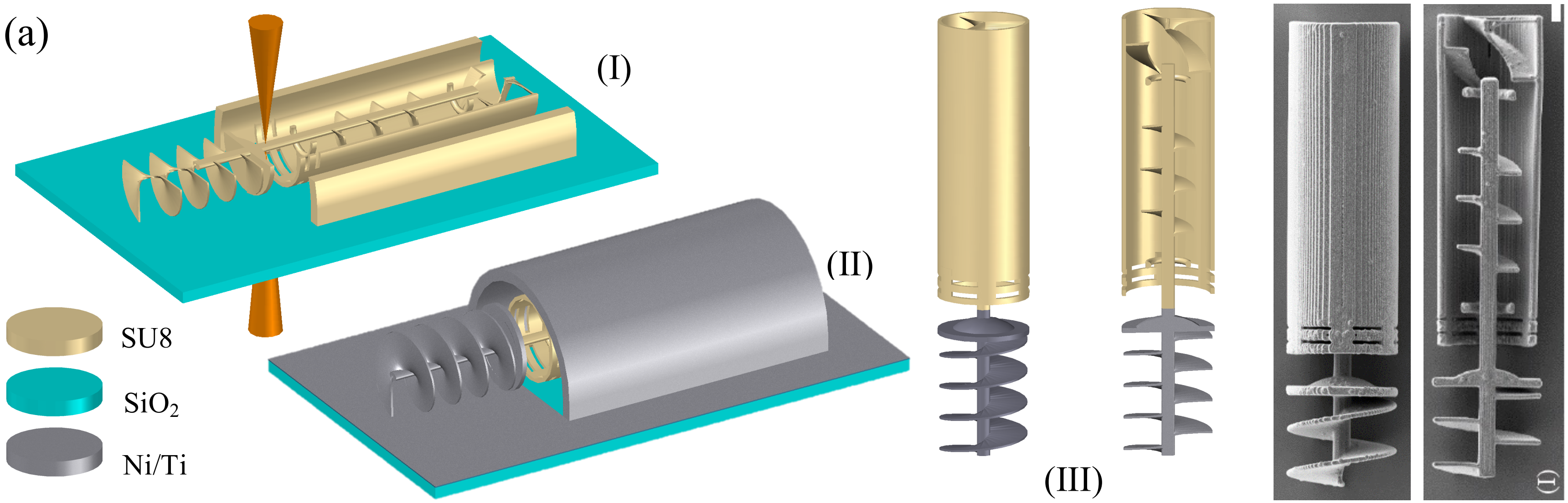
Mobile Fluidic Traps in 3D

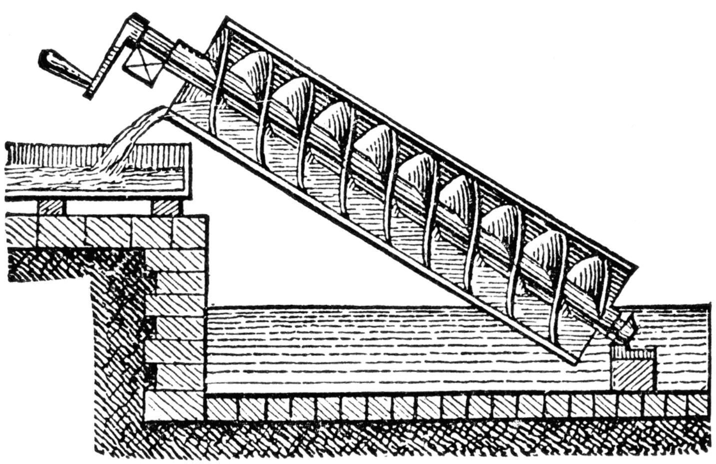




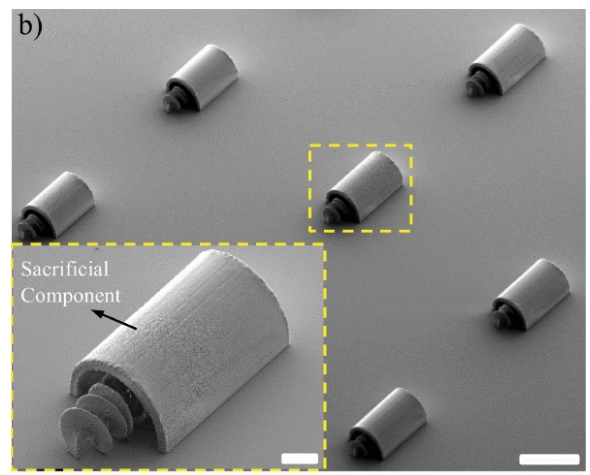


3D Printing Compound Machinery

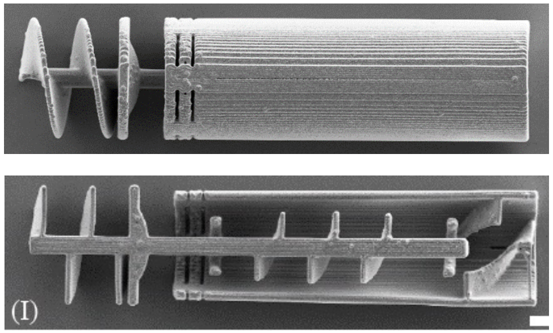


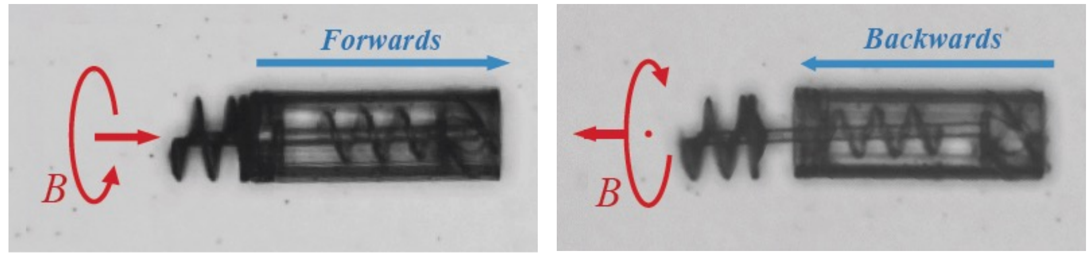


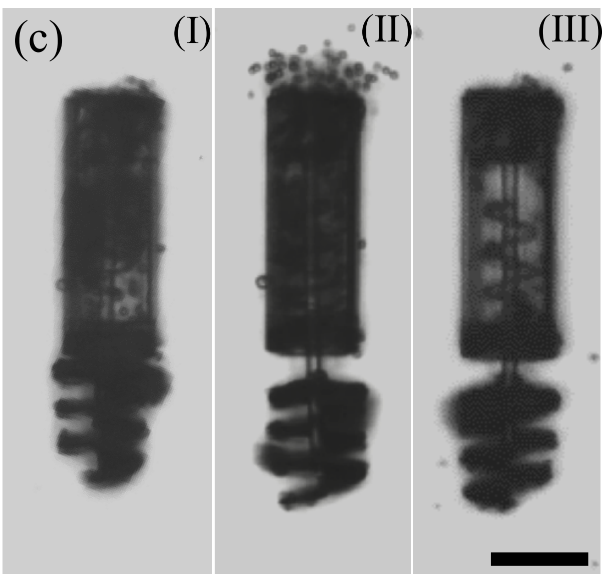
Archimedean Screw



Active Transport at Low Reynolds Number



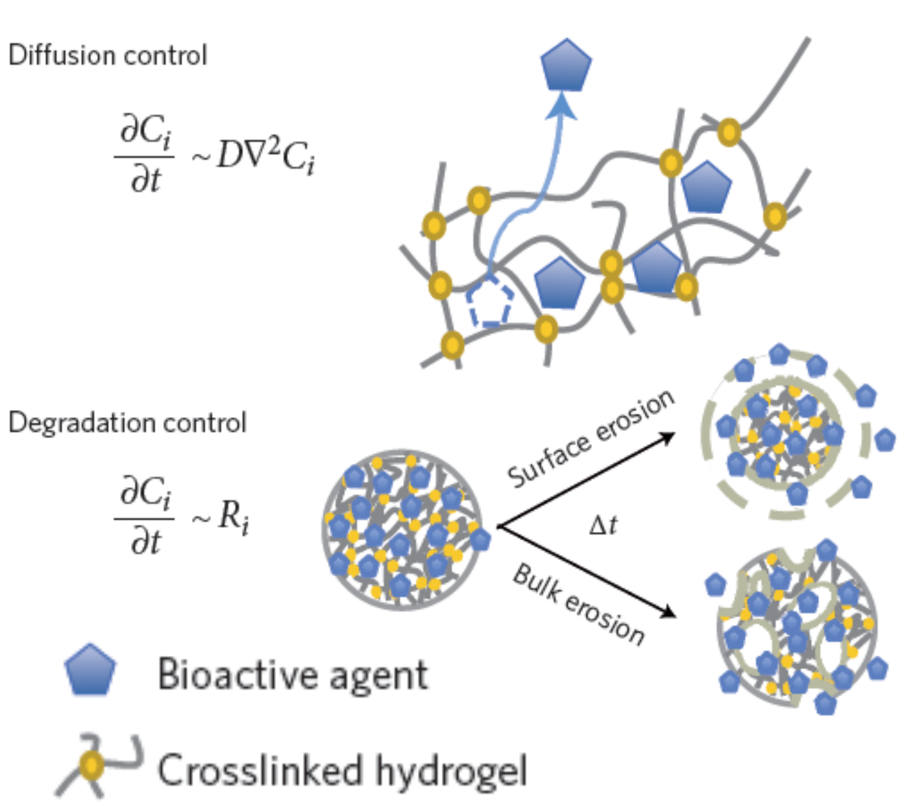




Passive vs Triggered Release

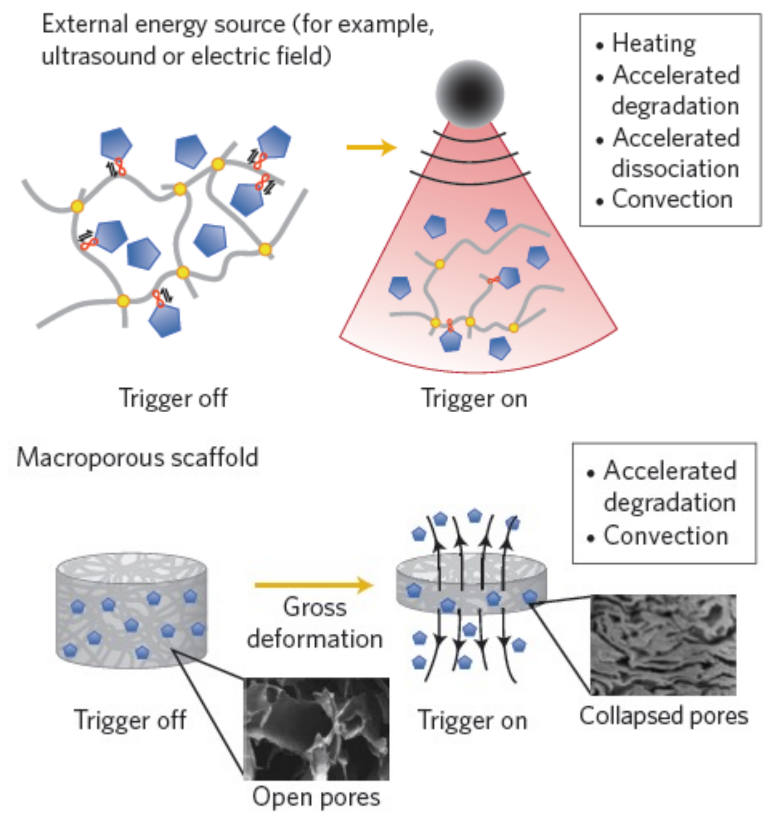
Mass balance/diffusion equation

$$\frac{\partial C_i}{\partial t} = \underbrace{D\nabla^2 C_i}_{\text{Diffusion}} - \underbrace{\nabla C_i v}_{\text{Convection}} + \underbrace{R_i}_{\text{Chemical reactions}}$$

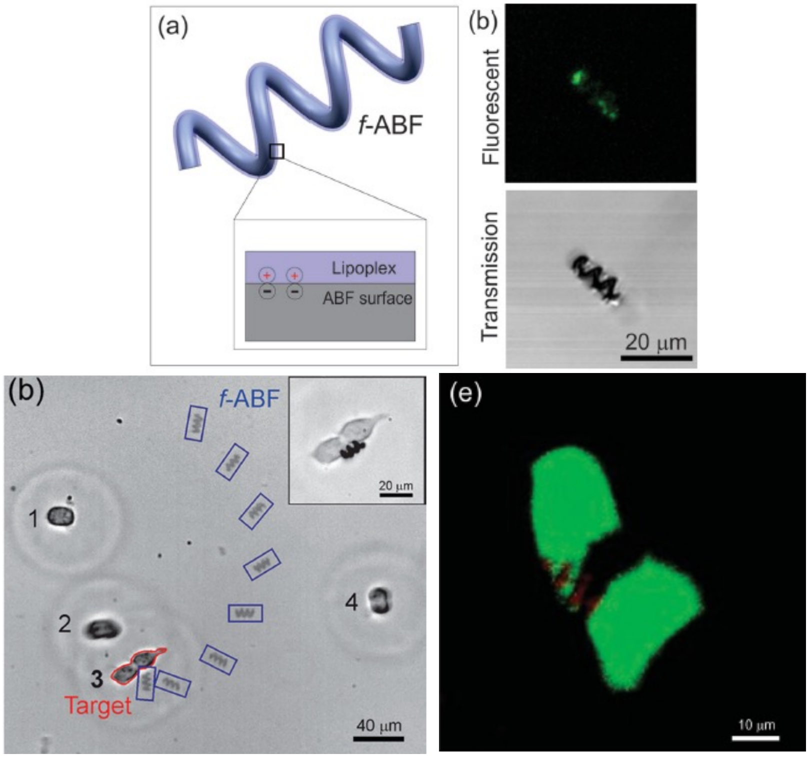


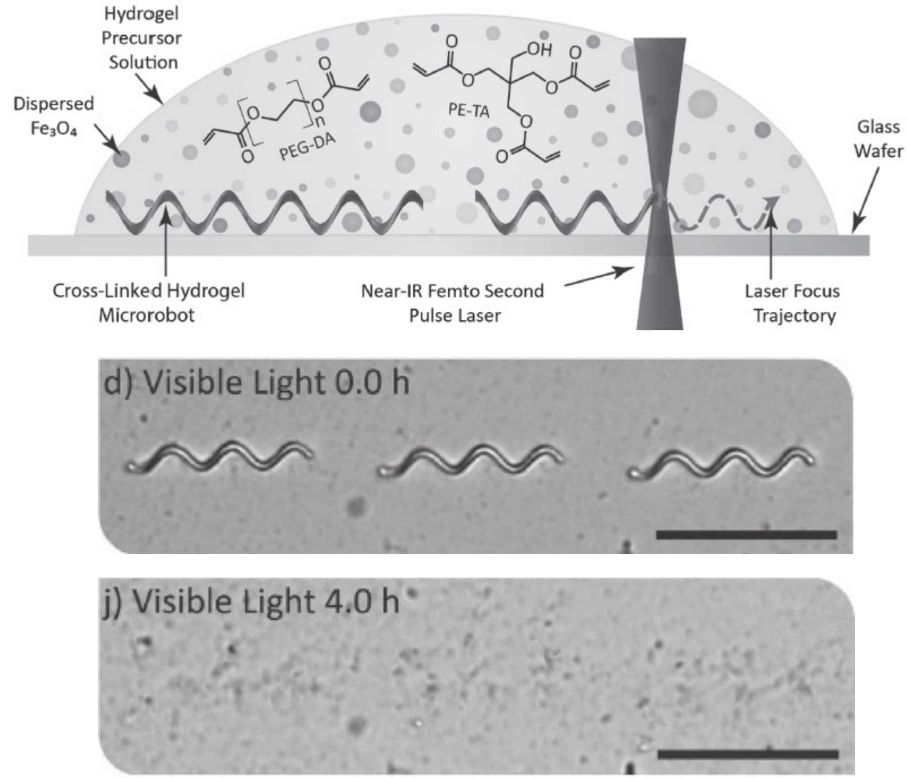
Mass balance/diffusion equation

$$\frac{\partial C_i}{\partial t} = D\nabla^2 C_i - \underbrace{\nabla C_i v}_{\text{Non-zero convective term}} + R_i$$



Passive Release of Encapsulated Payload





- Loading capacity: surface area vs volume
- One material for everything: form, magnetization, reservoir

Colloidal Self-Assembly of Magnetic Micromachines

- Viscous vs magnetic forces: Mason Number

$$\mathrm{Ma} = \frac{12^2 \eta \omega}{\mu_0 \mu_s M^2}$$

- Dynamics does not depend on volume fraction at low Mason number
- Critical Mason number
- Shape of the assembly depends on the frequency and field strength